Today’s Reading Assignment:
MIT 8.01 Course Notes
Chapter 9 Circular Motion Dynamics
Sections 9.1-9.2
Announcements

Problem Set 3 due Week 5 Tuesday at 9 pm in box outside 26-152

Math Review Week 5 Tuesday 9-11 pm in 26-152.

Next Reading Assignment (W04D3):
MIT 8.01 Course Notes
Chapter 9 Circular Motion Dynamics
Section 9.3
Circular Motion: Vector Description

Position
\[ \mathbf{r}(t) = r \mathbf{\hat{r}}(t) \]

Component of Angular Velocity
\[ \omega_z \equiv \frac{d\theta}{dt} \]

Velocity
\[ \mathbf{v} = v_\theta \mathbf{\hat{\theta}}(t) = r \left( \frac{d\theta}{dt} \right) \mathbf{\hat{\theta}} \]

Component of Angular Acceleration
\[ \alpha_z \equiv \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2} \]

Acceleration
\[ \mathbf{a} = a_r \mathbf{\hat{r}} + a_\theta \mathbf{\hat{\theta}} \]

\[ a_r = -r \left( \frac{d\theta}{dt} \right)^2 = -\left( \frac{v^2}{r} \right), \quad a_\theta = r \left( \frac{d^2\theta}{dt^2} \right) \]
Concept Question: Car in a Turn

You are a passenger in a racecar approaching a turn after a straight-away. As the car turns left on the circular arc at constant speed, you are pressed against the car door. Which of the following is true during the turn (assume the car doesn't slip on the roadway)?

1. A force pushes you away from the door.
2. A force pushes you against the door.
3. There is no force that pushes you against the door.
4. The frictional force between you and the seat pushes you against the door.
5. There is no force acting on you.
6. You cannot analyze this situation in terms of the forces on you since you are accelerating.
7. Two of the above.
8. None of the above.
Newton’s Second Law: Force and Kinematics

\[ \vec{F} = m \vec{a} \]

.physics = geometry

\[ \hat{r} : \quad F_r = -mr \left( \frac{d\theta}{dt} \right)^2 \]

\[ \hat{\theta} : \quad F_\theta = mr \frac{d^2\theta}{dt^2} \]
Concept Question: Circular Motion and Force

A pendulum bob swings down and is moving fast at the lowest point in its swing. $T$ is the tension in the string, $W$ is the gravitational force exerted on the pendulum bob. Which free-body diagram below best represents the forces exerted on the pendulum bob at the lowest point? The lengths of the arrows represent the relative magnitude of the forces.

- (a) $T = W$
- (b) $T = W$
- (c) $T < W$
- (d) $T > W$
- (e) $T < W$
Strategy: Applying Newton’s Second Law for Circular Motion

- Always has a component of acceleration pointing radially inward
- May or may not have tangential component of acceleration
- Draw Free Body Diagram for all forces
- $mv^2/r$ is not a force but mass times acceleration and does not appear on force diagram
- Choose a sign convention for radial and tangential directions and check that signs for forces and acceleration are consistent
A ball is suspended from a vertical rod by two strings of equal strength and equal length. The strings are very light and do not stretch. The rod is spun with a constant angular acceleration. Which string breaks first?

1. the upper string
2. the lower string
3. they break simultaneously
4. cannot tell without more information
Worked Example: String Theory

Choose cylindrical coordinates

Draw free-body force diagram

Apply Newton’s Second Law in Radial and vertical directions

\[ \hat{r} : -T_{\text{upper}} \sin 45^\circ - T_{\text{lower}} \sin 45^\circ = -ms \sin 45^\circ \omega^2 \quad \Rightarrow \quad T_{\text{upper}} + T_{\text{lower}} = ms \omega^2 \]

\[ \hat{k} : T_{\text{upper}} \cos 45^\circ - T_{\text{lower}} \cos 45^\circ - mg = 0 \quad \Rightarrow \quad T_{\text{upper}} - T_{\text{lower}} = \sqrt{2}mg \]

Solve for tensions in two strings:

Increase \( \omega \) then upper string breaks first.

\[ T_{\text{upper}} = m(s \omega^2 + \sqrt{2}g) / 2 \]

\[ T_{\text{lower}} = m(s \omega^2 - \sqrt{2}g) / 2 \]
A body of mass $m$ slides without friction on the inside of a cone. The axis of the cone is vertical and gravity is directed downwards. The apex half-angle of the cone is $\beta$ as shown in the figure. The path of the object happens to be a circle in a horizontal plane. The speed of the particle is $v_0$. Find the radius of the circular path and the time it takes to complete one circular orbit in terms of the given quantities and $g$. 
Demo: Rotating Bucket B104

A bucket of balls is spun in a circle. No balls spills out.
Worked Example: Rotating Bucket

A pail of mass $m_p$ is full of water (mass $m_w$). A string is attached to the handle of the pail which is then whirled around a vertical circle at constant speed $v$. You may assume that the center of mass of the bucket and the water undergoes circular motion with radius $R$. What is the minimum speed that the pail must have at the top of its circular motion if the water is not to spill out of the upside-down pail? For faster speeds, find the tension in the string and the magnitude of the contact force between the water and the bucket.

Gravitational force: \( \vec{F}_{1,2} \)
gravitational force on body 2 due to the interaction between bodies 1 and 2

Magnitude: \( |\vec{F}_{1,2}| = G \frac{m_1 m_2}{r_{12}^2} \)

\( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2} \)

Direction: Attractive
Worked Example: Moon’s Orbit

Calculate the period of the moon’s orbit around the earth. This is called the *sidereal month* because it is the time that it takes for the Moon to return to a given position with respect to the stars. Assume that the moon is only under the influence of the earth’s gravitational force and that the moon is moving with a constant speed in a circular orbit around the earth. The mass of the earth is \( m_e = 5.98 \times 10^{24} \text{ kg} \). The mass of the moon is \( m_m = 7.36 \times 10^{22} \text{ kg} \). The radius of the orbit is \( r_{e,m} = 3.8 \times 10^8 \text{ m} \).
Worked Example: Moon’s Orbit

Assume that the Moon’s Orbit is nearly circular. Choose polar coordinates in plane of orbit. Draw a free-body force diagram.

![Diagram showing Earth and Moon with forces and distances labeled]

Apply Newton’s Second Law: \( \vec{r} : - \frac{G m_e m_m}{r_{e,m}^2} = -m_m \frac{4\pi^2 r_{e,m}}{T^2} \)

Solve for period of orbit

\[
T = 2\pi \sqrt{\frac{r_{e,m}^3}{G m_e}}
\]

\[
T = 2\pi \sqrt{\frac{(3.82 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} = 2.35 \times 10^6 \text{ s}
\]

\[
T = (2.35 \times 10^6 \text{ s})(\frac{1\text{ day}}{8.64 \times 10^4 \text{ s}}) = 27.2 \text{ days}
\]
Discussion: Moon’s Orbit

The *synodic month* is longer than the average sidereal month because the Earth is traveling around the Sun. So the Moon must travel a little farther than one full circle around the Earth in order to be on the other side of the Earth from the Sun one month later.

At the next full moon, the moon has traveled an additional time equal to

\[ T / 12 = 2.25 \text{ days} \]

so the observed period is

\[ T_{\text{synodic}} = T_{\text{sidereal}} \left(1 + 1 / 12\right) \]

\[ = \left(13 / 12\right) T_{\text{sidereal}} = 29.45 \text{ days} \]
Table Problem: Experiment 1
Circular Motion

A small ball of mass $m$ is attached to one end of a spring with spring constant $k$ and unstretched length $r_0$. The other end of the spring is attached to the central axis of a motor. The motor rotates at a constant angular speed of magnitude $\omega$. The ball and spring rotate in a horizontal plane. You may neglect the gravitational force exerted on the ball. What is the radius of the orbit?
Experiment 1: Circular Motion