3-Dimensional Rotation: Gyroscopes

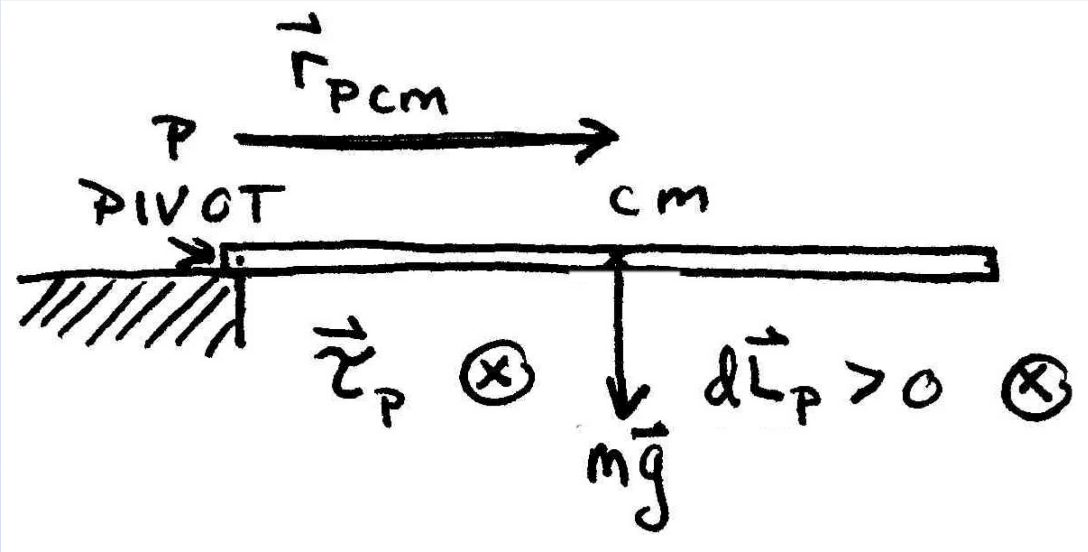
8.01

W13D2

Next Reading Assignment: W013D3

Quiz 9 Angular Momentum

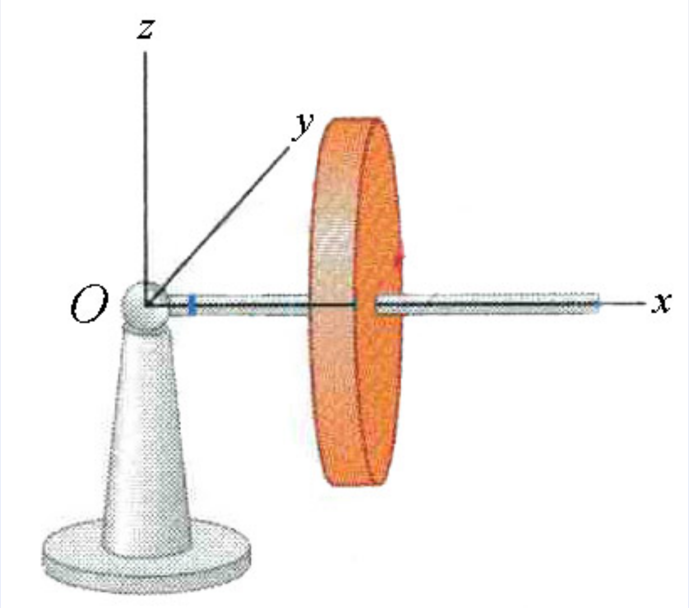
Mini Demo: Pivoted Falling Stick



Magnitude of the angular momentum about pivot changes.

Direction of change of angular momentum about pivot is the same as direction of angular momentum about pivot

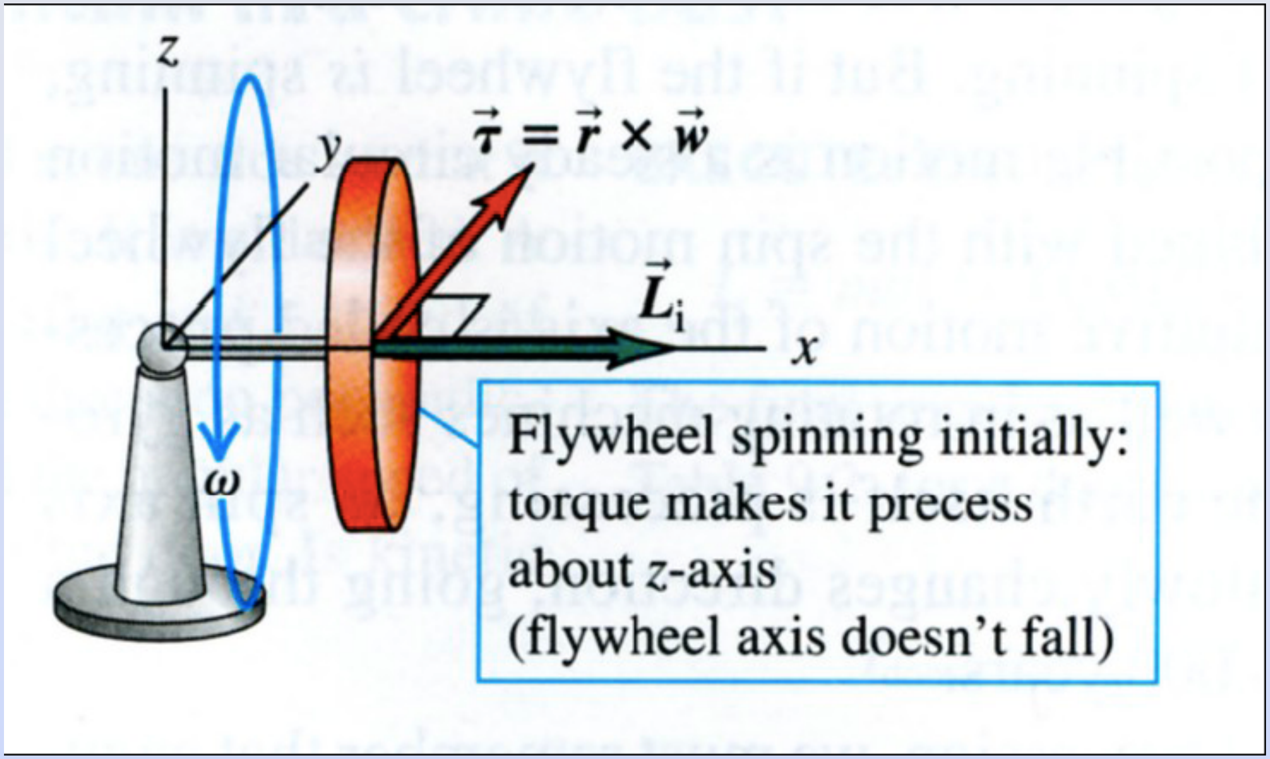
Mini Demo: Pivoted Unspinning Gyroscope



Magnitude of the angular momentum about pivot changes.

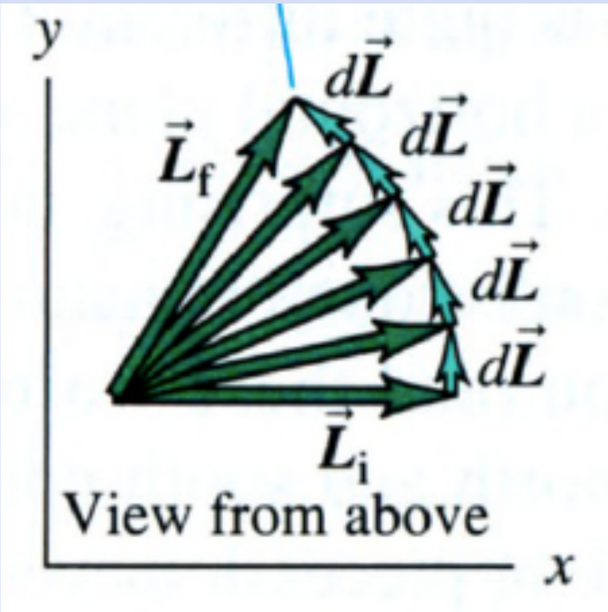
Direction of change of angular momentum about pivot is the same as direction of angular momentum about pivot

Torque on a Flywheel



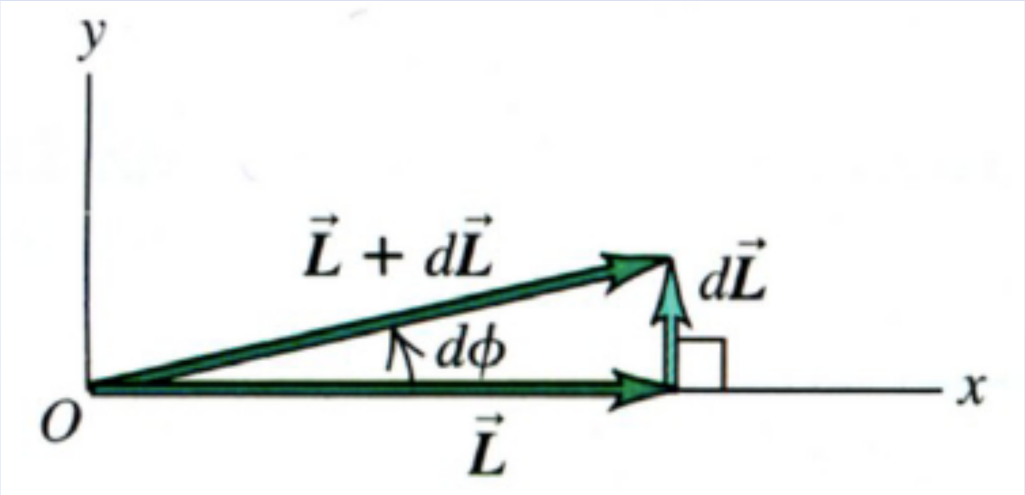
Torque on a Flywheel: Change in Angular Momentum

There is an initial angular momentum $\vec{\mathbf{L}}_i$; torque $\vec{\tau}$ only changes direction of $\vec{\mathbf{L}}$, ($d\vec{\mathbf{L}}$ vectors are perpendicular to $\vec{\mathbf{L}}$).



Torque on a Flywheel: Change in Angular Momentum

In a time dt, the angular momentum vector and the flywheel precess through an angle dϕ



$$\Omega = \frac{d\phi}{dt} = \frac{\left|d\vec{\mathbf{L}}\right| / \left|\vec{\mathbf{L}}\right|}{dt} = \frac{\left|\tau_y\right|}{\left|L_x\right|} = \frac{rmg}{I_{cm}\omega}$$

Concept Question

For the simple gyroscope problem we just solved, if the mass of the disk is doubled how will the new precession rate Ω be related to the original rate Ω_0?

1) $\Omega = 4\,\Omega_0$

2) $\Omega = 2\,\Omega_0$

3) $\Omega = \Omega_0$

4) $\Omega = (1/2)\,\Omega_0$

5) $\Omega = (1/4)\,\Omega_0$

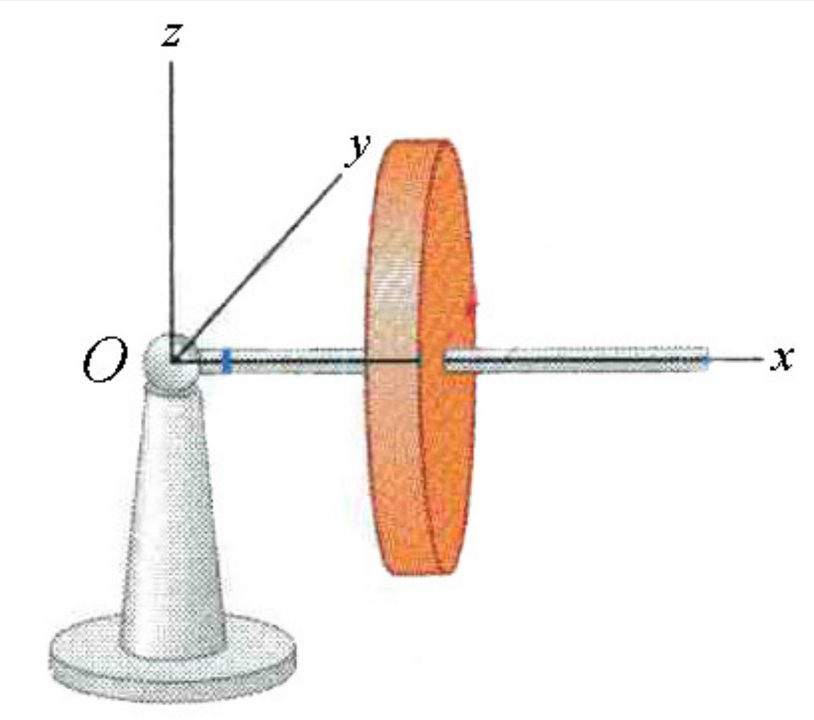


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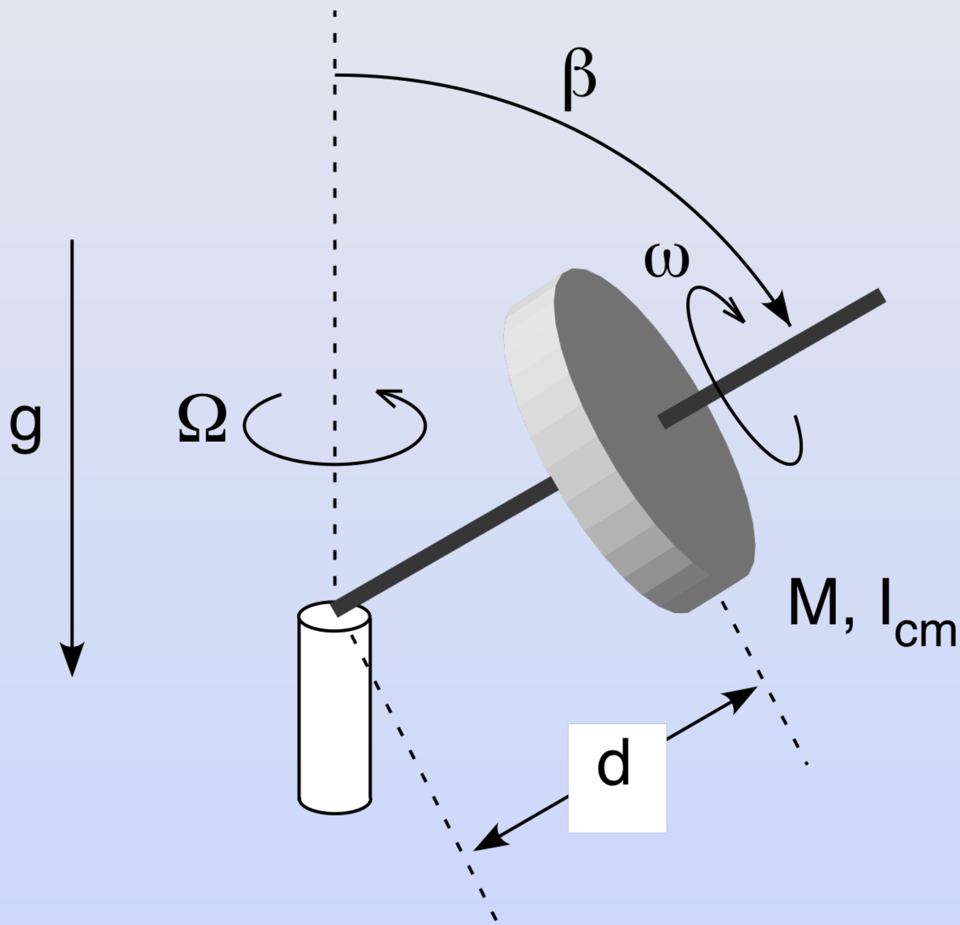
Table Problem: Gyroscope at an Angle



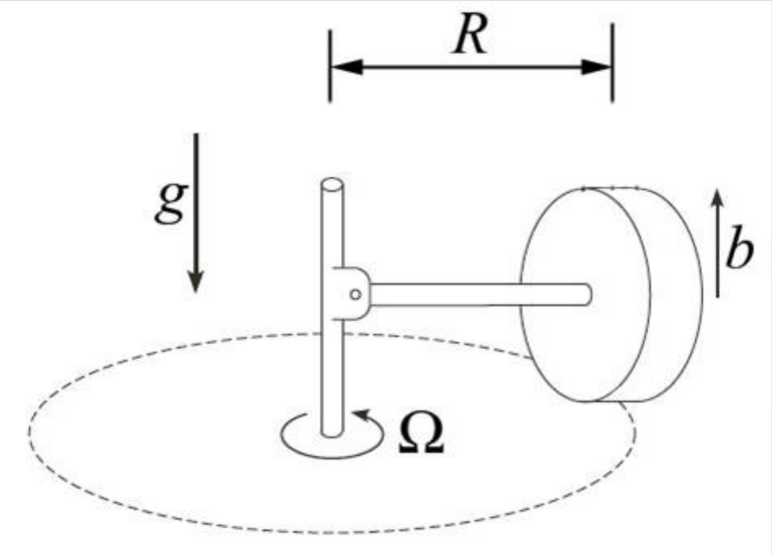
Find the magnitude and the sign of Ω in terms of the other parameters.

Mini-Experiment:

Toy gyroscopes

Table Problem: Mill Stone

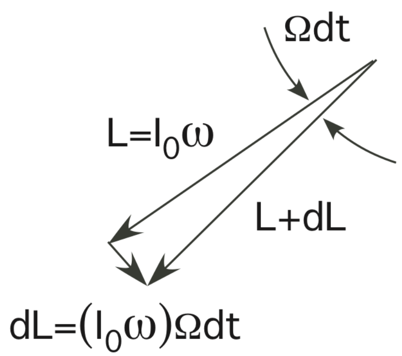
In a grain mill, grain is ground by a massive wheel which rolls without slipping in a circle on a flat horizontal mill stone driven by a vertical shaft. The rolling wheel has mass M, radius b and is constrained to roll in a horizontal circle of radius R at angular speed Ω. What value of Ω is necessary for the wheel to push down on the lower mill stone with a force equal to twice its weight. The mass of the axle of the wheel can be neglected.



Odd tables: Find the magnitude of dL/dt about the pivot in terms of M, R, b and Ω.

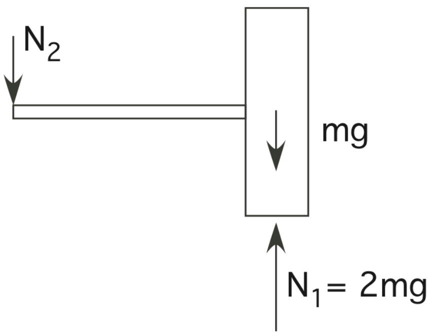
Even tables: Find the Find the magnitude of The torque about the pivot in terms of M, R, b and g.

Begin by examining the angular momentum and how it changes with time.



Viewed from the side away from the center post, the mill wheel is rotating clockwise; thus its angular momentum vector always points toward the post in a horizontal plane. The change in the angular momentum, $d\vec{L}$, is anti-parallel to the velocity of the center of mass of the wheel.

Use the rolling constraint on the wheel to obtain a relation between ω and Ω: $dx_{cm} = R\Omega dt = b\omega dt \quad \Rightarrow \quad \omega = (R/b)\Omega$. Now balance the forces on the wheel and its shaft in the vertical direction.



$$\cancelto{2mg}{N_1} - N_2 - mg = 0$$

$$\Rightarrow N_2 = mg$$

Referring to the diagram above one can see that the torque about the center of the wheel is anti-parallel to the velocity of the center of mass of the wheel and has a magnitude $\tau = N_2 R = mgR$.

$$\begin{aligned} \frac{dL}{dt} &= L\Omega = I\omega\Omega = (\frac{1}{2}mb^2)(\frac{R}{b})\Omega^2 \\ \text{also} &= \tau = mgR \\ &\Rightarrow \underline{\Omega = \sqrt{\frac{2g}{b}}} \end{aligned}$$

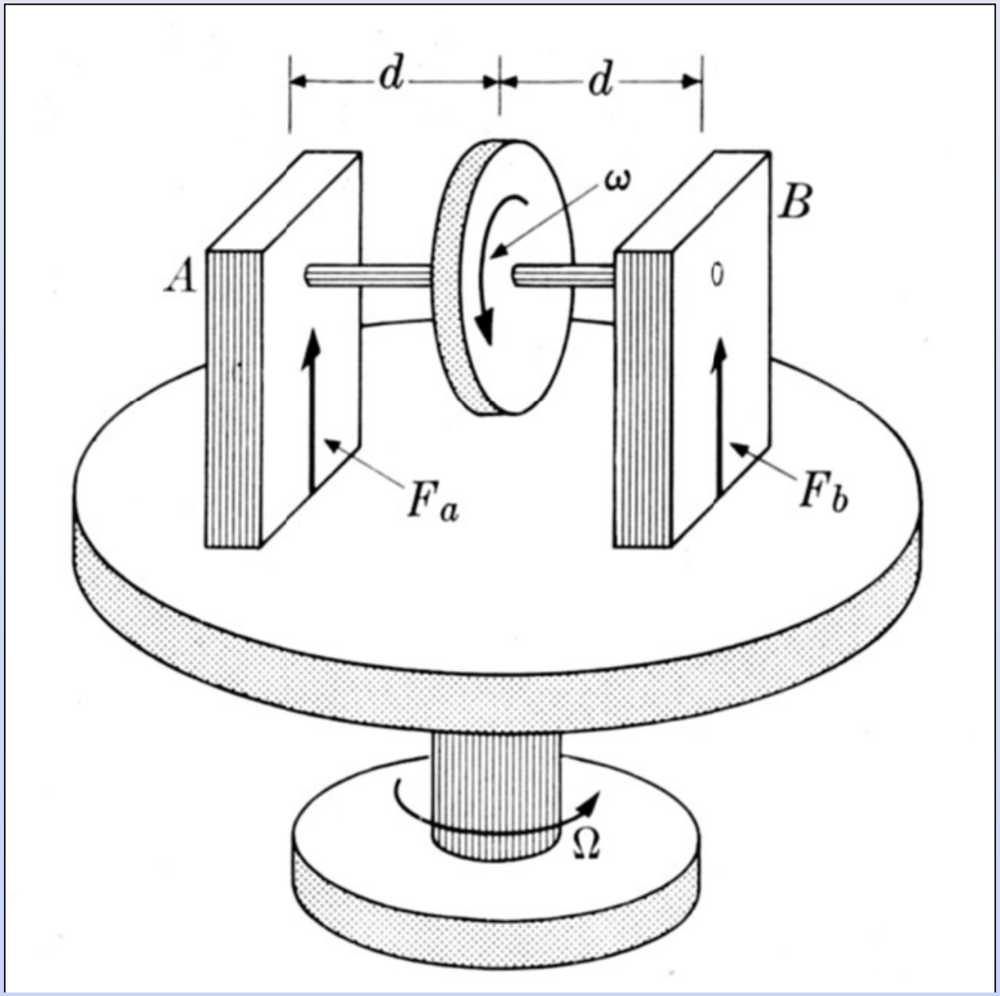
Table Problem: Gyroscope on rotating platform

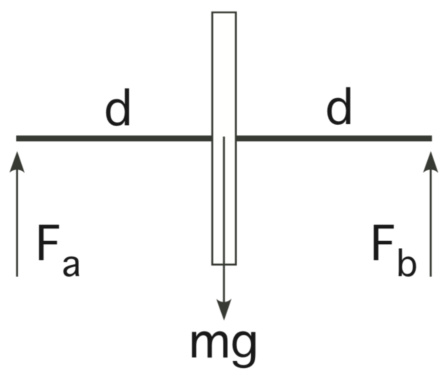
Odd tables

Find torque about the CM in terms of F_a, F_b and d.

Even tables

Find dL/dt about the CM in terms of I_{cm}, ω and Ω.





The vertical forces sum to zero since there is no vertical motion.

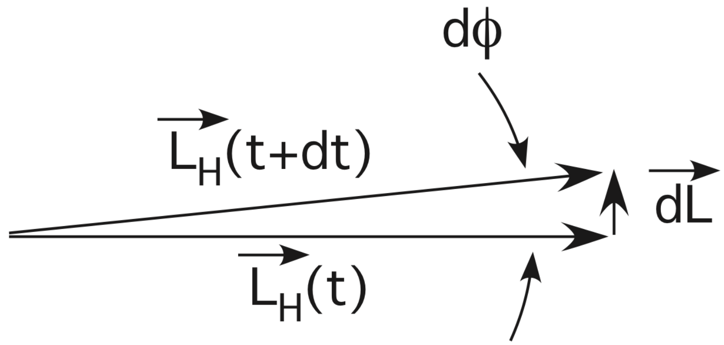
$$F_a + F_b = mg$$

Taking the direction into the board as being positive

$$\tau = F_a d - F_b d = (F_a - F_b)d$$

To complete the expression for the difference of the forces we need a value for τ.

Looking down on the gyroscope from above one has the following view. The $d\vec{L}$ in this diagram is in the same direction as the torque computed in the previous slide. $\vec{L_H}$ is the horizontal component of the angular momentum; the vertical component is not changeing with time.



$$
\begin{aligned}
|\vec{L_H}| &= I\omega \\
|d\vec{L}| &= |L_H| d\phi \\
\frac{|d\vec{L_H}|}{dt} &= |\vec{L_H}|\frac{d\phi}{dt} = I\omega\Omega
\end{aligned}
$$

$$
\begin{aligned}
F_a + F_b &= mg \\
F_a - F_b &= |\tau|/d = \frac{|d\vec{L}_H|}{dt}/d = I\omega\Omega/d
\end{aligned}
$$

$$
\begin{aligned}
F_a &= (1/2)(mg + I\omega\Omega/d) \\
F_b &= (1/2)(mg - I\omega\Omega/d)
\end{aligned}
$$

Note that if $\Omega = mgd/I\omega$, $F_b = 0$ and one could remove the right hand support. This is just the expression for simple gyroscopic motion.

The forces we just found are the forces that the mounts must apply to the gyroscope in order to cause it to move in the desired direction.

It is important to understand that the gyroscope is applying equal and opposite forces on the mounts, the structure that is holding it. This is a manifestation of “Newton’s third law force pairs”

Table Problem: Sopwith Camel

The Sopwith Camel was a single-engine fighter plane flown by British pilots during WWI (and also by the character Snoopy in the Peanuts comic strip). It was powered by a radial engine, and the entire engine rotated with the propeller. The Camel had an unfortunate property: if the pilot turned to the right the plane Tended to go into dive, while a left turn caused the plane to climb steeply. These tendencies caused inexperienced pilots to crash or stall during takeoff.

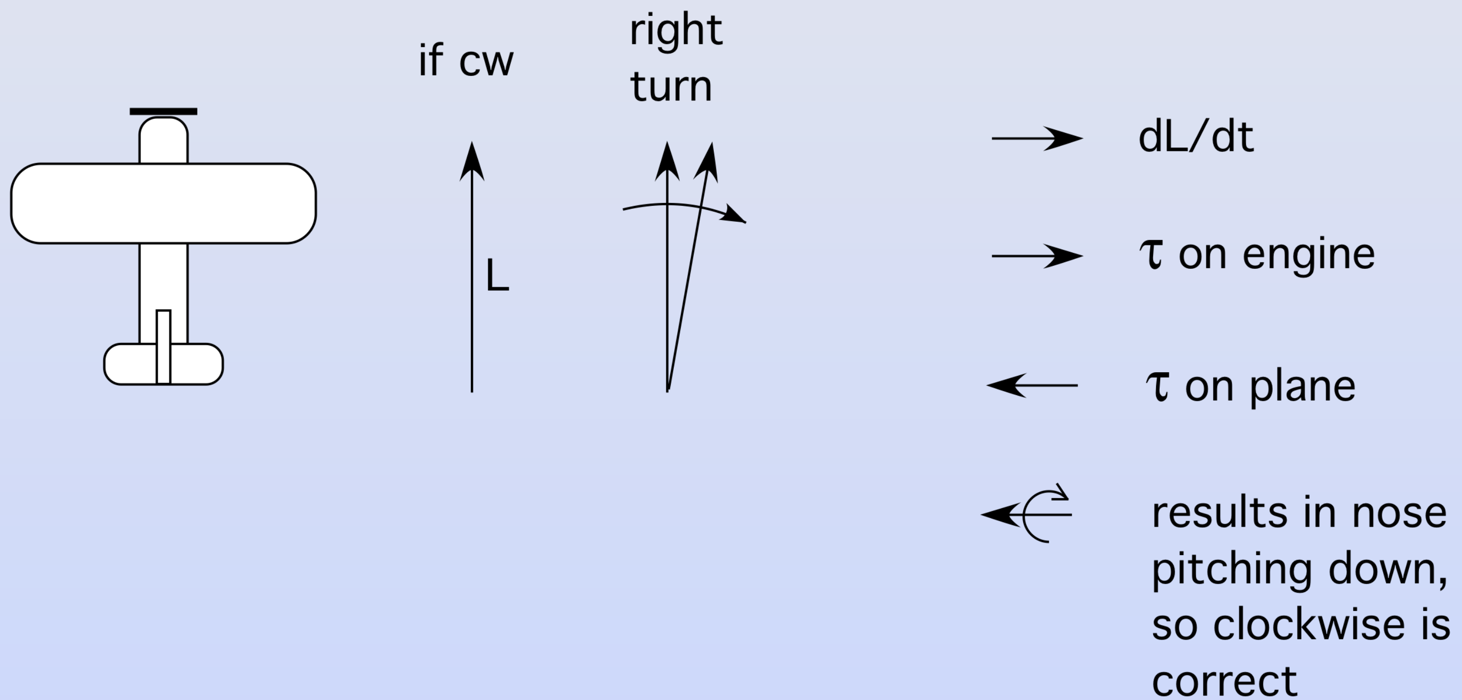
Table Problem: Sopwith Camel

From the perspective of the pilot, who sat behind the engine, did the engine rotate clockwise or counter clockwise?

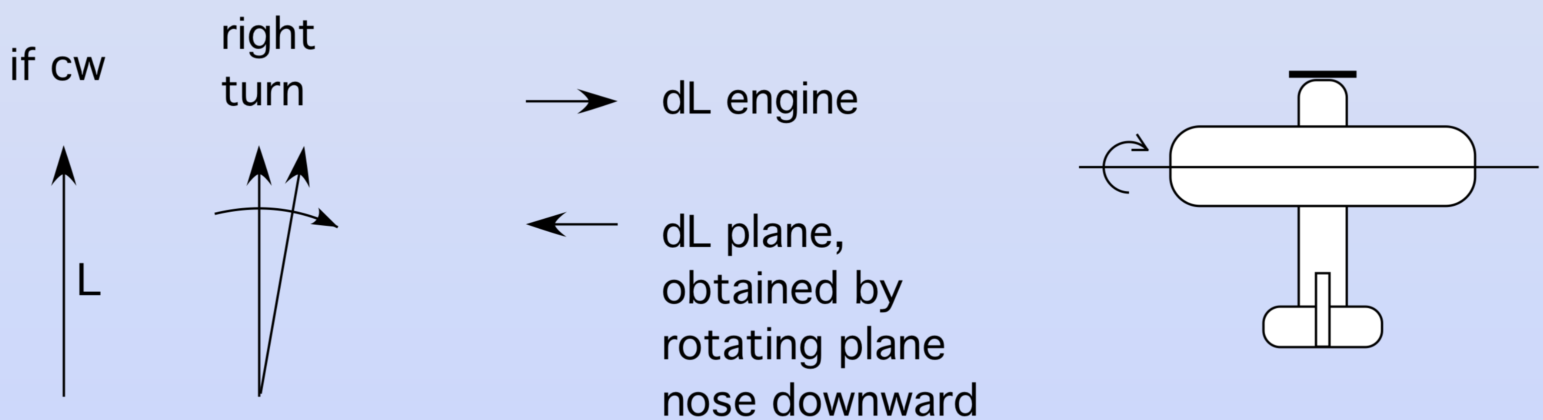
Odd tables: Argue on the basis of torque (and third law pairs).

Even tables: Argue on the basis of conservation of angular momentum in the horizontal plane.

Torque: Assume one of the two possible directions of rotation and see if it gives the correct result.



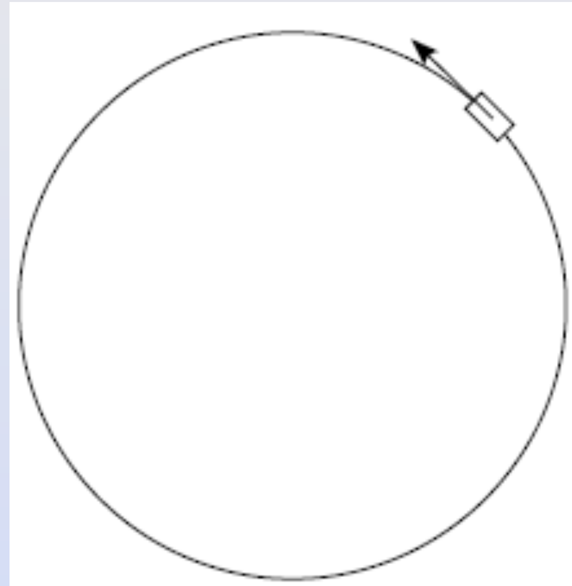
Angular momentum: The pilot begins turning right using the rudder on the tail section, applying an external torque changing $L_{z.}$ The large horizontal component of the engine's L swings right. If the pilot does not use the elevators to apply another external torque, the horizontal component of the plane's motion must counter the engine, to ensure no net change in horizontal angular momentum.



Concept Question

When making a turn every car has a tendency to roll over because its center of mass is above the plane where the wheels contact the road.

Imagine a race car going counter clockwise on a circular track. It could mitigate this effect by mounting a gyroscope on the car. To be effective the angular velocity vector of the gyro should point



1) ahead
2) behind
3) to the left
4) to the right
5) up
6) down