

## Kinematics and One Dimensional Motion

8.01  
W02D1

## Kinematics Vocabulary

- *Kinema* means movement
- Mathematical description of motion
  - Position
  - Time Interval
  - Displacement
  - Velocity; absolute value: speed
  - Acceleration
  - Averages of the later two quantities.

## Coordinate System in One Dimension

Used to describe the position of a point in space

A coordinate system consists of:

1. An origin at a particular point in space
2. A set of coordinate axes with scales and labels
3. Choice of positive direction for each axis: unit vectors
4. Choice of type: Cartesian or Polar or Spherical

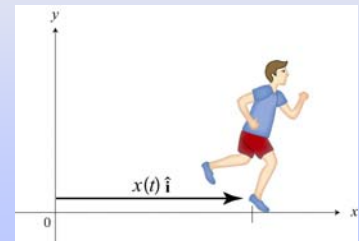
Example: Cartesian One-Dimensional Coordinate System



## Position

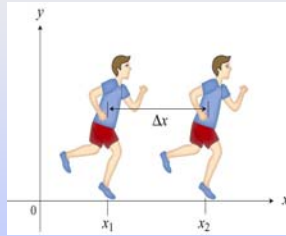
- A vector that points from origin to body.
- Position is a function of time
- In one dimension:

$$\vec{\mathbf{x}}(t) = x(t)\hat{\mathbf{i}}$$



## Displacement Vector

Change in position vector of the object during the time interval  $\Delta t = t_2 - t_1$



$$\Delta \vec{r} \equiv (x(t_2) - x(t_1)) \hat{i} \equiv \Delta x(t) \hat{i}$$

## Concept Question: Displacement

An object goes from one point in space to another. After the object arrives at its destination, the magnitude of its displacement is:

- 1) either greater than or equal to
- 2) always greater than
- 3) always equal to
- 4) either smaller than or equal to
- 5) always smaller than
- 6) either smaller or larger than

the distance it traveled.

## Average Velocity

The average velocity,  $\bar{v}(t)$ , is the displacement  $\Delta \vec{r}$  divided by the time interval  $\Delta t$

$$\bar{\mathbf{v}}(t) \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t} \hat{i} = \bar{v}_x(t) \hat{i}$$

The x-component of the average velocity is given by

$$\bar{v}_x(t) = \frac{\Delta x}{\Delta t}$$

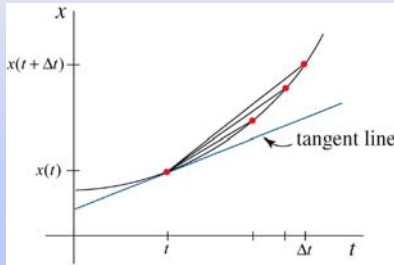
## Instantaneous Velocity

- For each time interval  $\Delta t$ , we calculate the x-component of the average velocity. As  $\Delta t \rightarrow 0$ , we generate a sequence of the x-component of the average velocities. The limiting value of this sequence is defined to be the x-component of the instantaneous velocity at the time  $t$ .

$$v_x(t) \equiv \lim_{\Delta t \rightarrow 0} \bar{v}_x = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \equiv \frac{dx}{dt}$$

## Instantaneous Velocity

x-component of the velocity is equal to the slope of the tangent line of the graph of x-component of position vs. time at time  $t$



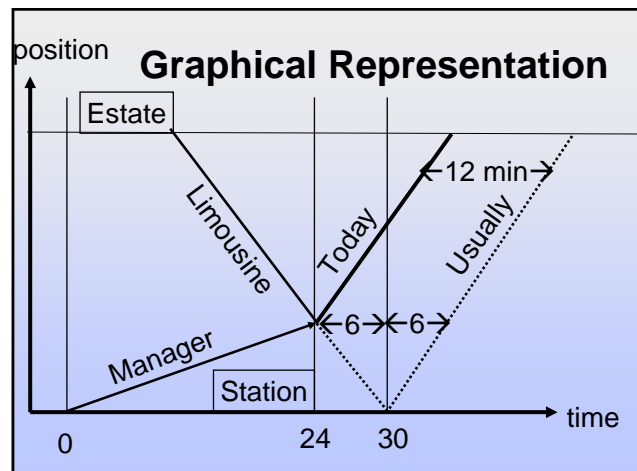
## Table Problem: Hedge Fund Ride Home

A hedge fund manager usually takes the train home and is met at the train station exactly at 6:30 by his chauffeur who drives him 6 miles to his estate. One day he leaves work early, arriving at the train station at 6:00. Finding his cell phone discharged, he jogs towards his home at 6 mph. After 24 minutes, he meets his chauffeur who turns around and takes him home.

- How much earlier than usual does he arrive home?
- What else can you determine from this information?

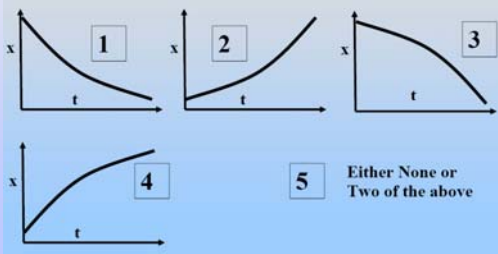
## Stuck?

- Represent the Problem in New Way
  - Graphical
  - Pictures with descriptions
  - Pure verbal
  - Equations
- Could You Solve it If?
  - The problem were simplified?
  - You knew some other fact/relationship
  - Solve any part of problem, even simple one



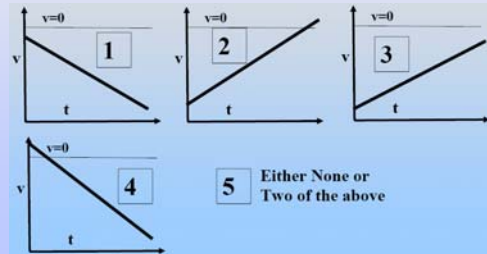
### CQ: Representing Motion - 1d

Which of these is a possible graph of the position,  $x(t)$ ?



### CQ: Representing Motion - 1d

Which of these is a possible graph of the x-component of velocity?



### Average Acceleration

Change in instantaneous velocity divided by the time interval  $\Delta t = t_2 - t_1$

$$\bar{\mathbf{a}} \equiv \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t} \hat{\mathbf{i}} = \frac{(v_{x,2} - v_{x,1})}{\Delta t} \hat{\mathbf{i}} = \frac{\Delta v_x}{\Delta t} \hat{\mathbf{i}} = \overline{a_x} \hat{\mathbf{i}}$$

The x-component of the average acceleration

$$\overline{a_x} = \frac{\Delta v_x}{\Delta t}$$

### Instantaneous Acceleration

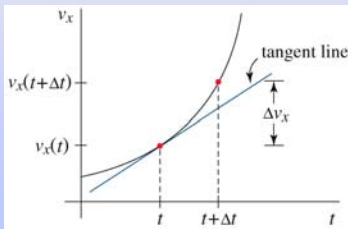
For each time interval  $\Delta t$ , we calculate the x-component of the average acceleration. As  $\Delta t \rightarrow 0$ , we generate a sequence of x-component of average accelerations. The limiting value of this sequence is defined to be the x-component of the instantaneous acceleration at the time  $t$ .

$$\bar{\mathbf{a}}(t) = a_x(t) \hat{\mathbf{i}} \equiv \lim_{\Delta t \rightarrow 0} \overline{a_x} \hat{\mathbf{i}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_x}{\Delta t} \hat{\mathbf{i}} = \lim_{\Delta t \rightarrow 0} \frac{v_x(t + \Delta t) - v_x(t)}{\Delta t} \hat{\mathbf{i}} \equiv \frac{dv_x}{dt} \hat{\mathbf{i}}$$

$$a_x(t) = \frac{dv_x}{dt}$$

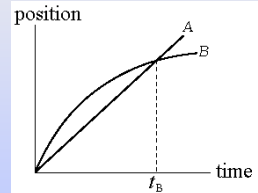
## Instantaneous Acceleration

The  $x$ -component of acceleration is equal to the slope of the tangent line of the graph of the  $x$ -component of the velocity vs. time at time  $t$



## Concept Question: One-Dimensional Kinematics

The graph shows the position as a function of time for two trains running on parallel tracks. For times greater than  $t=0$ , which is true:



1. At time  $t_B$ , both trains have the same velocity.
2. Both trains speed up all the time.
3. Both trains have the same velocity at some time before  $t_B$ .
4. Somewhere on the graph, both trains have the same acceleration.

## Summary: Constant Acceleration

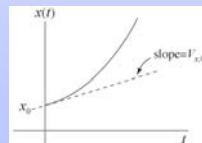
- Acceleration  $a_x = \text{constant}$



- Velocity  $v_x(t) = v_{x,0} + a_x t$



- Position  $x(t) = x_0 + v_{x,0}t + \frac{1}{2}a_x t^2$



## Example: Free Fall

- Choose coordinate system with  $x$ -axis vertical, origin at ground, and positive unit vector pointing upward.

- Acceleration:  $a_x = -g = -(9.8 \text{ m} \cdot \text{s}^{-2})$

$$v_x(t) = v_{x,0} - gt$$

- Velocity

- Position:  $x(t) = x_0 + v_{x,0}t - \frac{1}{2}gt^2$

### Concept Question: One-Dimensional Kinematics

You are throwing a ball straight up in the air. At the highest point, the ball's

- 1) velocity and acceleration are zero
- 2) velocity is nonzero but its acceleration is zero
- 3) acceleration is nonzero, but its velocity is zero
- 4) velocity and acceleration are both nonzero.

### Concept Question: One-Dimensional Kinematics

A person standing at the edge of a cliff throws one ball straight up and another ball straight down, each at the same initial speed. Neglecting air resistance, which ball hits the ground below the cliff with the greater speed:

1. ball initially thrown upward;
2. ball initially thrown downward;
3. neither; they both hit at the same speed.

### Problem Solving Strategies: One-Dimensional Kinematics

### Worked Example: Runner

A runner accelerates from rest with a constant x-component of acceleration  $2.0 \text{ m s}^{-2}$  for 2.0 s and then travels at a constant velocity for an additional 6.0 s. How far did the runner travel?

## I. Understand – get a conceptual grasp of the problem

Question 1: How many objects are involved in the problem?

Question 2: How many different stages of motion occur?

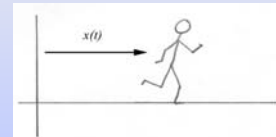
Question 3: For each object, how many independent directions are needed to describe the motion of that object?

Question 4: What choice of coordinate system best suits the problem?

Question 5: What information can you infer from the problem?

## I. Understand – get a conceptual grasp of the problem

- stage 1: constant acceleration
- stage 2: constant velocity.



Tools:  
Coordinate system  
Kinematic equations

## II. Devise a Plan

- Sketch the problem
- Choose a coordinate system
- Write down the complete set of equations for the position and velocity functions; identify any specified quantities; clean up the equations.
- Finding the “missing links”: count the number of independent equations and the number of unknowns.
- You can solve a system of  $n$  independent equations if you have exactly  $n$  unknowns.
- Look for constraint conditions

## II. Devise a Plan

Stage 1: constant acceleration

Initial conditions:  $x_0 = 0$   $v_{x,0} = 0$

Kinematic Equations:  $x(t) = \frac{1}{2}a_x t^2$   $v_x(t) = a_x t$

Final Conditions: end acceleration at  $t = t_a$

position:  $x_a \equiv x(t = t_a) = \frac{1}{2}a_x t_a^2$

velocity  $v_{x,a} \equiv v_x(t = t_a) = a_x t_a$

### III. Devise a Plan

Stage 2: constant velocity, time interval  $[t_a, t_b]$

- runs at a constant velocity for the time  $t_b - t_a$
- final position  $x_b \equiv x(t=t_b) = x_a + v_{x,a}(t_b - t_a)$

### III. Solve

- Design a strategy for solving a system of equations.
- Check your algebra and dimensions.
- Substitute in numbers.
- Check your results and units.

### III. Solve

- three independent equations

$$x_a = \frac{1}{2} a_x t_a^2$$

$$v_{x,a} = a_x t_a$$

$$x_b = x_a + v_{x,a}(t_b - t_a)$$

- Six symbols:  $x_b, x_a, v_{x,a}, a_x, t_a, t_b$

- Three given quantities specified in problem

$$a_x, t_a, t_b$$

### III. Solve

- Solve for distance the runner has traveled

$$x_b \equiv x(t=t_b) = \frac{1}{2} a_x t_a^2 + a_x t_a (t_b - t_a) = a_x t_a t_b - \frac{1}{2} a_x t_a^2$$



## IV. Review

- Check your results, do they make sense (think!)
- Check limits of an algebraic expression (be creative)
- Think about how to extend model to cover more general cases (thinking outside the box)
- Solved problems act as models for thinking about new problems. (Mechanics provides a foundation of solved problems.)

## IV. Review

Numerical results:

Runner accelerated for  $t_a = 2.0\text{ s}$

Initial acceleration:  $a_x = 2.0\text{ m}\cdot\text{s}^{-2}$

Runs at a constant velocity for  $t_b - t_a = 6.0\text{ s}$

Total time of running  $t_b = t_a + 6.0\text{ s} = 2.0\text{ s} + 6.0\text{ s} = 8.0\text{ s}$

Total distance running

$$x_{\text{total}} = a_x t_a t_b - \frac{1}{2} a_x t_a^2 = (2.0\text{ m}\cdot\text{s}^{-2})(8.0\text{ s})(8.0\text{ s}) - \frac{1}{2}(2.0\text{ m}\cdot\text{s}^{-2})(2.0\text{ s})^2 = 2.8 \times 10^1\text{ m}$$

Final velocity  $v_{x,a} = a_x t_a = (2.0\text{ m}\cdot\text{s}^{-2})(2.0\text{ s}) = 4.0\text{ m}\cdot\text{s}^{-1}$

## Table Problem: One Dimensional Kinematics

### Bus and car

At the instant a traffic light turns green, a car starts from rest with a given constant acceleration,  $5.0 \times 10^{-1}\text{ m}\cdot\text{s}^{-2}$ . Just as the light turns green, a bus, traveling with a given constant speed,  $1.6 \times 10^1\text{ m}\cdot\text{s}^{-1}$ , passes the car. The car speeds up and passes the bus some time later. How far down the road has the car traveled, when the car passes the bus?

## Summary: Time Dependent Acceleration

- Acceleration is a non-constant function of time

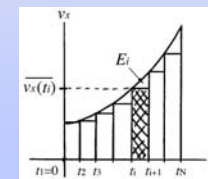
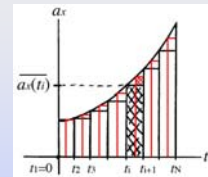
$$a_x(t)$$

- Change in velocity

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt'$$

- Change in position

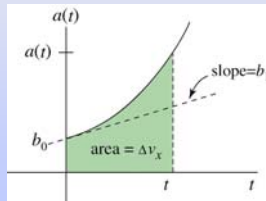
$$x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$



## Example: Non-constant acceleration

- Consider an object released at time  $t = 0$  with an initial x-component of velocity  $v_{x,0}$ , located at position  $x_0$ , and accelerating according to

$$a_x(t) = b_0 + b_1t + b_2t^2$$



- Find the velocity and position as a function of time.

## Example: Non-constant acceleration

Velocity:

$$\begin{aligned} v_x(t) &= v_{x,0} + \int_{t=0}^{t=t} a_x(t') dt' \\ &= v_{x,0} + \int_{t=0}^{t=t} (b_0 + b_1t' + b_2t'^2) dt' = v_{x,0} + b_0t' \Big|_{t=0}^{t=t} + \frac{1}{2}b_1t'^2 \Big|_{t=0}^{t=t} + \frac{1}{3}b_2t'^3 \Big|_{t=0}^{t=t} \\ &= v_{x,0} + b_0t + \frac{1}{2}b_1t^2 \end{aligned}$$

Position:

$$\begin{aligned} x(t) &= x_0 + \int_{t=0}^{t=t} v_x(t') dt' = \\ &= x_0 + \int_{t=0}^{t=t} (v_{x,0} + b_0t' + \frac{1}{2}b_1t'^2 + \frac{1}{3}b_2t'^3) dt' = v_{x,0}t + \frac{1}{2}b_0t^2 + \frac{1}{6}b_1t^3 + \frac{1}{12}b_2t^4 \end{aligned}$$