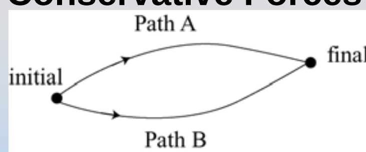
Potential Energy and Conservation of Energy

8.01

W08D2

Conservative Forces



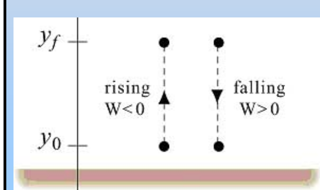
Definition: Conservative Force If the work done by a force in moving an object from an initial point to a final point is independent of the path (A or B),

$$W_c \equiv \int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}} \qquad \text{(path independent)}$$

then the force is called a *conservative force* which we denote by $\vec{\mathbf{F}}_c$

Example: Gravitational Force

- Consider the motion of an object under the influence of a gravitational force near the surface of the earth
- The work done by gravity depends only on the change in the vertical position

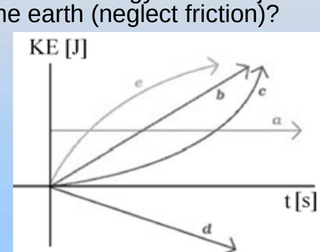


$$\boxed{W_g = F_g \Delta y = -mg\Delta y}$$

Concept Question: Energy

An object is dropped to the earth from a height of 10m. Which of the following sketches best represent the kinetic energy of the object as it approaches the earth (neglect friction)?

1. a
2. b
3. c
4. d
5. e



Change in Potential Energy

Definition: Change in Potential Energy The *change in potential energy* of a body associated with a conservative force $\vec{\mathbf{F}}_c$ is the negative of the work done by the conservative force in moving the body along any path connecting the initial and the final positions.

$$\boxed{\Delta U \equiv -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}} = -W_c}$$

Work-Energy Theorem

The work done by the total force in moving an object from A to B is equal to the change in kinetic energy

$$W^{\text{total}} \equiv \int_{z_0}^{z_f} \vec{\mathbf{F}}^{\text{total}} \cdot d\vec{\mathbf{r}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \equiv \Delta K$$

When the only forces acting on an object are **conservative forces**

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_c$$

then the change in potential energy is

$$\Delta U = -W_c = -W^{\text{total}}$$

Therefore $-\Delta U = \Delta K$

Conservation of Energy for Conservative Forces

When the only forces acting on an object are conservative

$$\Delta K + \Delta U = 0$$

Definition: Mechanical Energy The *mechanical energy* is the sum of the kinetic and potential energies

$$E^{\text{mechanical}} \equiv K + U$$

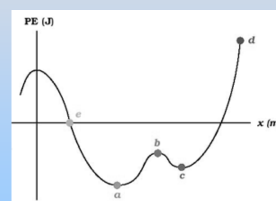
Equivalently, the mechanical energy remains constant in time

$$E_f^{\text{mechanical}} = K_f + U_f = K_i + U_i = E_i^{\text{mechanical}}$$

Concept Question: Energy

Consider the following sketch of potential energy for a particle as a function of position. (There are no other non-conservative forces acting on the particle i.e. no dissipative forces or internal sources of energy.)

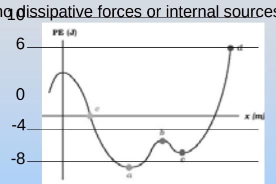
If a particle travels through the entire region of space shown in the diagram, at which point is the particle's velocity a maximum?



1. a
2. b
3. c
4. d
5. e

Concept Question: Energy

Consider the following sketch of potential energy for a particle as a function of position. (There are no other non-conservative forces acting on the particle i.e. no dissipative forces or internal sources of energy.)



What is the minimum total mechanical energy that the particle can have if you know that it has traveled over the entire region of X shown?

1. -8
2. 6
3. 10
4. It depends on direction of travel
5. Can't say - Potential Energy uncertain by a constant

Change in PE: Constant Gravity

Force: $\vec{\mathbf{F}}_{\text{grav}} = m\vec{\mathbf{g}} = F_{\text{grav},y}\,\hat{\mathbf{j}} = -mg\,\hat{\mathbf{j}}$

Work: $W_{\text{grav}} = F_{\text{grav},y}\Delta y = -mg\Delta y$

Potential Energy: $\Delta U = -W_{\text{grav}} = mg\Delta y = mg\left(y_f - y_0\right)$

Choice of Zero Point: Whatever "ground" is convenient

Concept Question: Work

Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

1. four times as small.
2. three times as small.
3. half as small.
4. the same.
5. undetermined-it depends on the time taken.

Concept Question: Energy and Choice of System

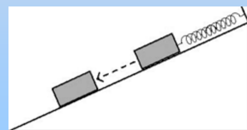
You lift a ball at constant velocity from a height h_i to a greater height h_f. Considering the ball and the earth together as the system, which of the following statements is true?

1. The potential energy of the system increases.
2. The kinetic energy of the system decreases.
3. The earth does negative work on the system.
4. You do negative work on the system.
5. Two of the above.
6. None of the above.

Concept Question: Energy and Choice of System

A block of mass *m* is attached to a relaxed spring on an inclined plane. The block is allowed to slide down the incline, and comes to rest. The coefficient of kinetic friction of the block on the incline is μ_k. For which definition of the system is the change in total energy (after the block is released) zero?

1. block
2. block + spring
3. block + spring + incline
4. block + spring + incline + Earth



Change in Potential Energy: Inverse Square Gravity

Force: $\vec{\mathbf{F}}_{m_1,m_2} = -\dfrac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

Work done: $W = \int_{r_0}^{r_f} \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int_{r_0}^{r_f}\left(-\dfrac{Gm_1m_2}{r^2}\right)dr = \left.\dfrac{Gm_1m_2}{r}\right|_{r_0}^{r_f} = Gm_1m_2\left(\dfrac{1}{r_f}-\dfrac{1}{r_0}\right)$

Potential Energy Change: $\Delta U_{\text{grav}} = -W_{\text{grav}} = -Gm_1m_2\left(\dfrac{1}{r_f}-\dfrac{1}{r_0}\right)$

Zero Point: $U_{\text{grav}}(r_0=\infty)=0$

Potential Energy Function: $U_{\text{grav}}(r) = -\dfrac{Gm_1m_2}{r}$

Change in PE: Spring Force

Force: $\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} = -kx\,\hat{\mathbf{i}}$

Work done: $W_{\text{spring}} = \int_{x=x_0}^{x=x_f}(-kx)\,dx = -\dfrac{1}{2}k\left(x_f^{\,2}-x_0^{\,2}\right)$

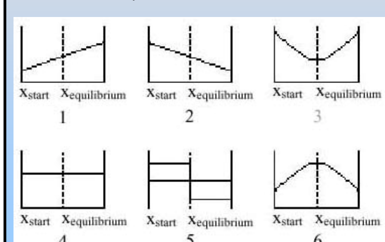
Potential Energy Change: $\Delta U_{\text{spring}} = -W_{\text{spring}} = \dfrac{1}{2}k\left(x_f^{\,2}-x_0^{\,2}\right)$

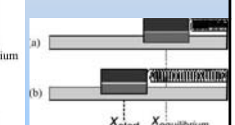
Zero Point: $U_{\text{spring}}(x=0)=0$

Potential Energy Function: $U_{\text{spring}}(x) = \dfrac{1}{2}kx^2$

Concept Question: Spring

In part (a) of the figure, a cart attached to a spring rests on a frictionless track at the position $x_{equilibrium}$ and the spring is relaxed. In (b), the cart is pulled to the position x_{start} and released. It then oscillates about $x_{equilibrium}$. Which graph correctly represents the potential energy of the spring as a function of the position of the cart?





Force and Potential Energy

In one dimension, the potential difference is

$$U(x) = U(x_0) - \int_A^B F_x \, dx$$

Force is the derivative of the potential energy

$$\boxed{F_x = -\frac{dU}{dx}}$$

Examples: (1) Spring Potential Energy: $U_{\text{spring}}(x) = \frac{1}{2}kx^2$

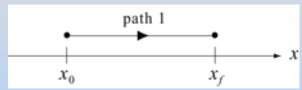
$$F_{x,spring} = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

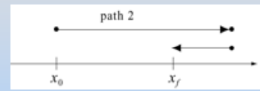
(2) Gravitational Potential Energy: $U_{\text{grav}}(r) = -\frac{Gm_1m_2}{r}$

$$F_{r,gravity} = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{Gm_1m_2}{r}\right) = -\frac{Gm_1m_2}{r^2}$$

Non-Conservative Forces

Work done on the object by the force depends on the path taken by the object





Example: friction on an object moving on a level surface

$$F_{\text{friction}} = \mu_k N$$

$$W_{\text{friction}} = -F_{\text{friction}} \Delta x = -\mu_k N \Delta x < 0$$

Non-Conservative Forces

Definition: Non-conservative force Whenever the work done by a force in moving an object from an initial point to a final point depends on the path, then the force is called a *non-conservative force*.

Change in Energy for Conservative and Non-conservative Forces

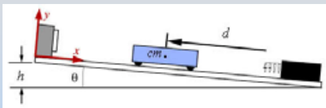
Total force: $\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_c^{\,total} + \vec{\mathbf{F}}_{nc}^{\,total}$

Total work done is change in kinetic energy:

$$W_{total} = \int_A^B \vec{\mathbf{F}}^{total} \cdot d\vec{\mathbf{r}} = \int_A^B \left(\vec{\mathbf{F}}_c^{\,total} + \vec{\mathbf{F}}_{nc}^{\,total}\right) \cdot d\vec{\mathbf{r}} = -\Delta U^{total} + W_{nc} = \Delta K$$

Energy Change: $\Delta K + \Delta U^{total} = W_{nc}$

Table Problem: Experiment 4 Cart-Spring on an Inclined Plane



An object of mass *m* slides down a plane that is inclined at an angle θ from the horizontal. The object starts out at rest. The center of mass of the cart is a distance *d* from an unstretched spring with spring constant *k* that lies at the bottom of the plane.

a) Assume the inclined plane to be frictionless. How far will the spring compress when the mass first comes to rest?

b) Now assume that the inclined plane has a coefficient of kinetic friction μ. How far will the spring compress when the mass first comes to rest? How much energy has been transformed into heat due to friction?

Experiment 4: Conservation of Energy