W11D2 PRS Concept Questions solutions

Concept Question: Figure Skater

A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her rotational inertia and her angular speed increases so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be

1. the same.
2. larger.
3. smaller

Answer: 2.

Rotational kinetic energy is \( K_{\text{rot}} = \frac{1}{2} I_{\text{cm}} \omega^2 \). Substituting \( L \), the magnitude of the angular momentum is \( |L_{\text{cm}}| = I_{\text{cm}} \omega \). Thus the rotational kinetic energy is \( K_{\text{rot}} = \frac{1}{2} |L_{\text{cm}}| \omega \). We know (a) that \( L \) is constant because there are no external torques acting on this isolated system and (b) that \( \omega \) increases. Therefore \( K_{\text{rot}} \) must increase. The "extra" energy comes from the work she does on her arms: She has to pull them in against their tendency to continue in a straight line, thereby doing work on them.
**Concept Question: Radial Force**

A tetherball of mass $m$ is attached to a post of radius $R$ by a string. Initially it is a distance $r_0$ from the center of the post and it is moving tangentially with a speed $v_0$. The string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole. Ignore gravity. Until the ball hits the post,

1. The energy and angular momentum about the center of the post are constant.

2. The energy of the ball is constant but the angular momentum about the center of the post changes.

3. Both the energy and the angular momentum about the center of the post, change.

4. The energy of the ball changes but the angular momentum about the center of the post is constant.

**Answer: 4**

The tension force points radially in; so torque about central point is zero: angular momentum about central point is constant.

$$\vec{r}_S = \vec{r}_{S,T} \times \vec{T} = \vec{0}$$

$$\vec{L}_{S,f}^{\text{system}} = \vec{L}_{S,0}^{\text{system}}$$
For all radial forces, (example gravitation), the angular momentum about the central point is a constant of the motion.

A small displacement of the ball has a radially component inward so the work done by tension is not zero.,

\[ dW_{ext} = \mathbf{T} \cdot d\mathbf{r} \neq 0. \]

Hence the mechanical energy is not constant.
Concept Question: Non-radial force:

A tetherball of mass $m$ is attached to a post of radius $R$ by a string. Initially it is a distance $r_0$ from the center of the post and it is moving tangentially with a speed $v_0$. The string wraps around the outside of the post. Ignore gravity. Until the ball hits the post,

1. The energy and angular momentum about the center of the post are constant.

2. The energy of the ball is constant but the angular momentum about the center of the post changes.

3. Both the energy of the ball and the angular momentum about the center of the post, change.

4. The energy of the ball changes but the angular momentum about the center of the post is constant.

**Answer 2.**

The tension force points towards contact point; so torque about central point is not zero: angular momentum about central point is not constant.
Small displacement is always perpendicular to string since at each instant in time ball undergoes instantaneous circular motion about string contact point with pole.

Therefore the tension force is perpendicular to the displacement

\[ dW^{ext} = \mathbf{T} \cdot d\mathbf{r} = 0. \]

Hence mechanical energy is constant.
Concept Question: Streetcar

A streetcar is freely coasting (no friction) around a large circular track. It is then switched to a small circular track.

When coasting on the smaller circle the streetcar's

1. mechanical energy is conserved and angular momentum about the center is constant
2. mechanical energy is not conserved and angular momentum about the center is constant
3. mechanical energy is not conserved and angular momentum about the center is not constant
4. mechanical energy is conserved and angular momentum about the center is not constant.

Answer: 4.

Carefully draw a free body diagram for the streetcar while it is on the link of track connecting the two circular tracks. In the vertical direction, the force of gravity at all times balances the vertical component of the normal force of the track on the car, so there is no acceleration in the vertical direction and we may ignore it for the rest of this problem. Along the plane of motion, the wheels are guided by component of the normal force in the plane. This force is always normal to the direction of motion, so no work is done by this force. Therefore the streetcar's total energy does not change, and its speed remains constant. Note that this force does exert a torque around the center of the circles because it does not point radially inward, and therefore its angular momentum is not constant!