Concept Question: Which of the following functions $x(t)$ has a second derivative which is proportional to the negative of the function $\frac{d^2x}{dt^2} = -x$?

1. $x(t) = \frac{1}{2} at^2$
2. $x(t) = Ae^{t/T}$
3. $x(t) = Ae^{-t/T}$
4. $x(t) = A\cos\left(\frac{2\pi}{T} t\right)$

Answer 4: The first derivative of $x(t) = A\cos\left(\frac{2\pi}{T} t\right)$ is given by:

$$\frac{dx(t)}{dt} = -\left(\frac{2\pi}{T}\right) A\sin\left(\frac{2\pi}{T} t\right).$$

The second derivative is:

$$\frac{d^2x(t)}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 A\cos\left(\frac{2\pi}{T} t\right) = -\left(\frac{2\pi}{T}\right)^2 x(t).$$

Concept Question:

The first derivative $v_x = dx/dt$ of the sinusoidal function $x = A\cos\left(\frac{2\pi}{T} t\right)$ is:

1. $v_x(t) = A\cos\left(\frac{2\pi}{T} t\right)$
2. $v_x(t) = -A\sin\left(\frac{2\pi}{T} t\right)$
3. $v_x(t) = -\frac{2\pi}{T} A\sin\left(\frac{2\pi}{T} t\right)$
4. $v_x(t) = \frac{2\pi}{T} A\cos\left(\frac{2\pi}{T} t\right)$

Answer 3:

Concept Question: A block of mass $m$ is attached to a spring with spring constant $k$ is free to slide along a horizontal frictionless surface. At $t = 0$ the block-spring system is stretched an amount $x_0 > 0$ from the equilibrium position and is released from rest. What is the $x$-component of the velocity of the block when it first comes back to the
equilibrium position?

1. $v_x = -x_0 \frac{T}{4}$
2. $v_x = x_0 \frac{T}{4}$
3. $v_x = -\sqrt{\frac{k}{m}} x_0$
4. $v_x = \sqrt{\frac{k}{m}} x_0$

Answer 3: The initial energy is the potential energy $E_0 = \frac{1}{2} k x_0^2$. The final energy is kinetic energy $E_f = \frac{1}{2} mv^2$. Setting these equal and solving for the velocity component $v_x = \pm \sqrt{\frac{k}{m}} x_0$. We choose the negative root because the block is moving in the negative $x$-direction.

Concept Question:

The position of a particle is given by

$$x(t) = D \cos(\omega t) - D \sin(\omega t), \quad D > 0$$

Where was the particle at $t = 0$?

1) 1
2) 2
3) 3
4) 4
5) 5
6) 1 or 5
7) 2 or 4
**Answer 4**: At $t = 0$ the particle has position $x(t = 0) = D\cos(0) - D\sin(0) = D > 0$. So the particle is located at either 4 or 5. The $x$-component of the velocity of the particle is given by $v_x(t) = \frac{dx(t)}{dt} = -\omega D\sin(\omega t) - D\omega \cos(\omega t)$, $D > 0$. At $t = 0$, the $x$-component of the velocity is $v_x(t = 0) = -\omega D\sin(0) - D\omega \cos(0) = -D\omega < 0$, is non-zero and negative so the particle cannot be at position 5 where it has zero velocity. Hence it must be at position 4.

**Concept Question**: A particle with total energy $E$ with $x > 0$ at $t = 0$

1) escapes
2) approximates simple harmonic motion
3) oscillates around a
4) oscillates around b
5) periodically revisits a and b
6) not enough information

Answer 1: Escapes to negative infinity because the kinetic energy is always positive or for all values of $x$ that are less than the value of $x$ on the right of the figure where the energy is equal to the potential energy.
**Concept Question:** A particle with total energy $E$ with $x > 0$ at $t = 0$

![Potential Energy Diagram](image)

1) escapes
2) approximates simple harmonic motion
3) oscillates around a
4) oscillates around b
5) periodically revisits a and b
6) not enough information

Answer 5: The particle oscillates, the potential is not quadratic so it is not simple harmonic. For the regions surrounding a and b, the kinetic energy is positive so the particle can be found at some times at a and b.

**Concept Question:** A particle with total energy $E$ with $x > 0$ at $t = 0$

![Potential Energy Diagram](image)
1) escapes
2) approximates simple harmonic motion
3) oscillates around a
4) oscillates around b
5) periodically revisits a and b
6) not enough information

Answer 6: The particle will either oscillate around a or b but we don’t know where it started. So we don’t have enough information.

**Concept Question:** A particle with total energy $E$ with $x > 0$ at $t = 0$

1) escapes
2) approximates simple harmonic motion
3) oscillates around a
4) oscillates around b
5) periodically revisits a and b
6) not enough information

Answer 3: The particle will now oscillate around a but the potential is still not quadratic so it will not be simple harmonic motion.
Concept Question: A particle with total energy $E$ with $x > 0$ at $t = 0$

1) escapes
2) approximates simple harmonic motion
3) oscillates around a
4) oscillates around b
5) periodically revisits a and b
6) not enough information

Answer 2: For Energies which are very close to the minimum of the potential energy, the potential energy can be approximated as a quadratic function and so the particle will undergo simple harmonic motion about a.