Massachusetts Institute of Technology
Department of Physics

8.01X

Fall 2001

Practice Final Exam SOLUTIONS

Problem 1: (*Experiment AM*)

In experiment AM, a circular steel washer, of mass m_1 and moment of inertia $I_w = 4.0 \times 10^{-5}$ kg-m^2, is mounted on the shaft of a small motor. Assume that the moment of inertia of the motor is negligible. The washer is set into motion. When it reaches 200 rad/sec, at t = 0 the power to the motor is shut off, and the washer slows down until it reaches an angular velocity of 140 rad/sec at t = 6 sec. At that moment a second identical circular steel washer of mass $m_2 = m_1$ is dropped on the first. The collision takes place over a time 0.2 sec.

a) What is the initial angular momentum of the washer at t=0?

b) What is the initial kinetic energy of the washer at t=0?

c) What is the magnitude of the frictional torque acting from t = 0 to 6 sec?

d) What is the angular velocity of both washers at t= 6.2 sec?

e) What is the average torque which the dropped washer exerts on the spinning washer during the time between 6.0 sec and 6.2 sec?

a) $L_0 = I_{cm}\,\omega_0 = (4.0 \times 10^{-5}\,kg\text{-}m^2)\left(200\,\frac{rad}{sec}\right) = 8.0 \times 10^{-3}\,\frac{kg\text{-}m^2}{sec}$

b) $K_0 = \frac{1}{2} I_{cm}\,\omega_0^2 = \frac{1}{2}(4.0 \times 10^{-5}\,kg\text{-}m^2)\left(2.0 \times 10^2\,\frac{rad}{sec}\right)^2 = 8.0 \times 10^{-1}\,J$

c) $\tau = I_{cm}\left|\frac{\Delta\omega}{\Delta t}\right| = (4.0 \times 10^{-5}\,kg\text{-}m^2)\left(\frac{140\,\frac{rad}{sec} - 200\,\frac{rad}{sec}}{6\,sec}\right) = 4 \times 10^{-4}\,J$

d) $-\tau\,\Delta t = \Delta L = 2 I_{cm}\,\omega_f - I_{cm}\,\omega_b$

$$\omega_f = \frac{-\tau\,\Delta t + I_{cm}\,\omega_b}{2 I_{cm}} = \frac{(-4 \times 10^{-4}\,J)(.2\,sec) + (4.0 \times 10^{-5}\,kg\text{-}m^2)\left(140\,\frac{rad}{sec}\right)}{(2)(4.0 \times 10^{-5}\,kg\text{-}m^2)}$$

$\omega_f = 69\,\frac{rad}{sec}$

e) $-\tau^{total}\,\Delta t = I_{cm}(\omega_f - \omega_b)$

$$-\tau^{total} = \frac{I_{cm}(\omega_f - \omega_b)}{\Delta t} = \frac{(4 \times 10^{-5}\,kg\text{-}m^2)\left(69\,\frac{rad}{sec} - 140\,\frac{rad}{sec}\right)}{.2\,sec} = -1.42 \times 10^{-2}\,J$$

$\tau^{total} = \tau_{frictional} + \tau_{dropped\ washer}$

$\tau_{dropped\ washer} = \tau^{total} - \tau_{f.} = -1.42 \times 10^{-2}\,J - -4 \times 10^{-4}\,J = -1.38 \times 10^{-2}\,J$

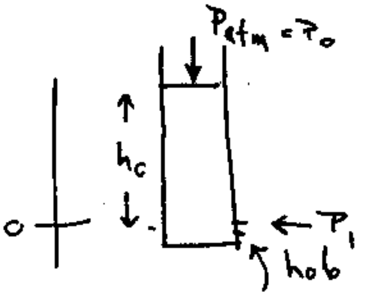
Problem 2: *(Bernoulli's Principle)*

A water bottle of area $A = 3.8 \times 10^{-3}$ m^2 containing water at atmospheric pressure is sealed shut with an initial volume of air $V_0 = 1.5 \times 10^{-4}$ m^3 and brought to the top of Mount Washington where the pressure is 0.88 atm. A hole is punched in the side of the bottle a distance $h_0 = 10.0$ cm below the water level. Assume the flow is steady out of the hole, and ignore any effects due to the hole. Note that 1 atm = 1.013×10^5 N/m^2.

a) What is the magnitude of the velocity of the efflux through the hole immediately after the hole is punched?

b) Assume the air is an ideal gas, what is the air pressure in the bottle as a function of the height h of the water level above the hole?

c) What is the magnitude of the velocity of the efflux through the hole as a function of the height h of the water level above the hole?



The pressure inside the bottle is atmosphere pressure

$P_0 = 1\text{ atm} = 1.013 \times 10^5\ N/m^2$

while the pressure outside the hole is the pressure atop Mt. Washington $P_1 = .88\text{ atm} = .89 \times 10^5\ N/m^2$

Bernoulli's Principle states that

$$P_0 + \rho g h_0 + \tfrac{1}{2}\rho v_0^2 = P_1 + \rho g h_1 + \tfrac{1}{2}\rho v_1^2$$

when the hole is first punched, $v_0 \approx 0$, $h_1 = 0$

so

$$P_0 + \rho g h_0 = P_1 + \tfrac{1}{2}\rho v_1^2$$

$$v_1 = \left(\frac{2(P_0 - P_1)}{\rho} + 2 g h_0\right)^{1/2}$$

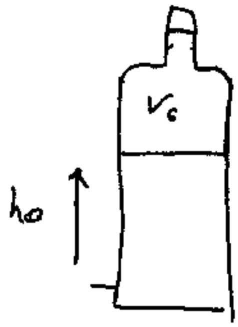
$$= \left(\frac{(2)(.12\text{ atm})(1.013 \times 10^5 \frac{Pa}{atm})}{10^3\ kg/m^3} + (2)(9.8\tfrac{m}{s^2})(.1\ m)\right)^{1/2}$$

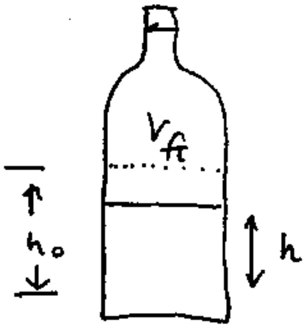
$$= 5.1\ m/s$$

Once the hole is punched and the water level starts dropping, the pressure inside drops.

If we assume an ideal gas law holds

$$P_0 V_0 = P_{ft} V_{ft}$$





let A = cross sectional area of bottle

$V_{ft} = A(h_0 - h) + V_0$, volume when water is at height h

$P_{ft} = P_0 \dfrac{V_0}{V_f} = \dfrac{P_0 V_0}{A(h_0-h)+V_0}$, pressure inside bottle when water is at height h

Bernoulli's equation becomes

$$\frac{P_0 V_0}{A(h_0-h)+V_0} + \rho g h = P_1 + \tfrac{1}{2}\rho v_1^2$$

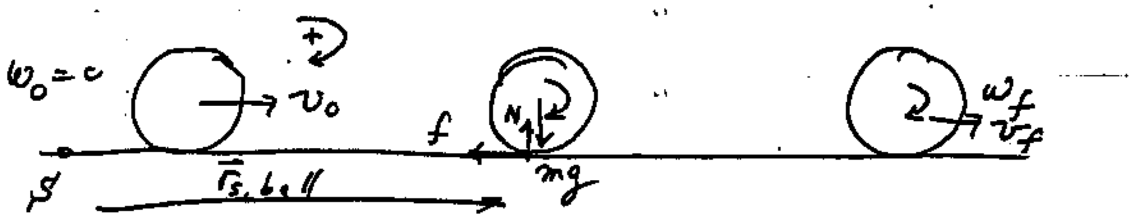
$$\left(\frac{2\left(P_0\left(\dfrac{V_0}{A(h_0-h)+V_0}\right) - P_1\right)}{\rho} + g h \right)^{1/2} = v_1$$

Problem 3: *(Torque and Conservation of Angular Momentum)* A bowling ball of mass m and radius R is thrown down an alley with an initial velocity v_0 and no initial angular velocity. The moment of inertia of the ball about the center of mass is $I = 2/5\ mR^2$. The coefficient of friction between the ball and the alley is μ.

a) What is the angular velocity of the ball when the ball just starts to roll without slipping down the alley?

b) What is the kinetic energy of the ball when the ball just starts to roll without slipping down the alley?

c) What is the change in kinetic energy ?



choose any point S along the alley. Since

$\vec{\tau}_S = \vec{r}_{S,ball} \times (m\vec{g} + \vec{N} + \vec{f}) = \vec{r}_{S,ball} \times \vec{f}$ since

$m\vec{g} = -\vec{N}$ so those torques cancel.

Since $\vec{r}_{S,ball}$ is antiparallel to $\vec{f}$,

$\vec{r}_{S,ball} \times \vec{f} = 0$. Thus $\vec{L}_S \equiv$ constant

$\vec{L}_0 = \vec{r}_{S,cm} \times m\vec{v}_0 = Rmv_0 \otimes$ into page

$\vec{L}_f = I_{cm}\,\omega_f \underset{\text{into page}}{\otimes} + \vec{r}_{S,cm} \times m\vec{v}_f$

$= (I_{cm}\,\omega_f + Rmv_f) \otimes$ into page

thus the rolling without slipping condition

$v_f = R\omega_f$

$L_f = I_{cm}\,\omega_f + RmR\omega_f = L_0 = Rmv_0$

$\frac{2}{5}mR^2\omega_f + mR^2\omega_f = Rmv_0$

$\frac{7}{5}mR^2\omega_f = Rmv_0 \Rightarrow \omega_f = \frac{5}{7}\frac{v_0}{R}$

$v_f = \frac{5}{7}v_0$

b) the kinetic energy of the ball

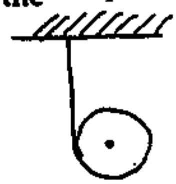
$$K = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 = \frac{1}{2} m v_f^2 + \frac{1}{2}\frac{2}{5} m R^2 \left(\frac{5}{7}\frac{v_0}{R}\right)^2$$

$$K = \frac{1}{2} m \left(\frac{5}{7} v_0\right)^2 + \frac{5}{49} m v_0^2 = \frac{5}{14} m v_0^2$$

$$\Delta K = K_0 - K_f = \frac{1}{2} m v_0^2 - \frac{5}{14} m v_0^2 = \frac{2}{7} m v_0^2$$

Problem 4: (*Torque and angular acceleration*)

A pulley of mass m, radius R, and moment of inertia $I = 1/2\ mR^2$ about the center of mass is hung from a ceiling with a massless string that winds around the rim of the pulley. At time $t = 0$ the pulley is released from rest.



(a) Draw the free body diagram on the pulley.

(b) Write down the equation describing the motion of the center of mass.

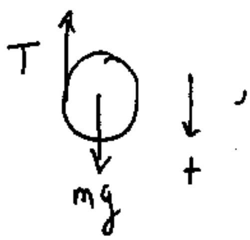
(c) Write down the rotational equation of motion.

(d) Find the direction and magnitude of the translational acceleration of the pulley.

(e) How long does it take for the pulley to fall a distance L?

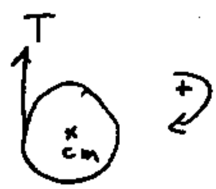
(f) What is the tension on the string?

a)



b) $mg - T = ma$

c)



$\tau =$	$I_{cm}\alpha$
$RT =$	$I_{cm}\alpha$

d) $T = \frac{I_{cm}\alpha}{R}$, $\alpha = \frac{a}{R}$ $\Rightarrow$ $T = \frac{1}{2}mR^2 \frac{a}{R\,R} = \frac{1}{2}ma$

Then $mg - T = ma \Rightarrow mg - \frac{1}{2}ma = ma \Rightarrow$

$mg = \frac{3}{2}ma \Rightarrow a = \frac{2}{3}g$

e) $L = \frac{1}{2}at^2 \Rightarrow t = \left(\frac{2L}{a}\right)^{1/2} = \left(\frac{2L}{2/3\,g}\right)^{1/2} = \left(\frac{3L}{g}\right)^{1/2}$

f) $T = \frac{1}{2}ma = \frac{1}{2}m\left(\frac{2}{3}g\right) = \frac{1}{3}mg$

Problem 5: (*Conservation of angular momentum*)

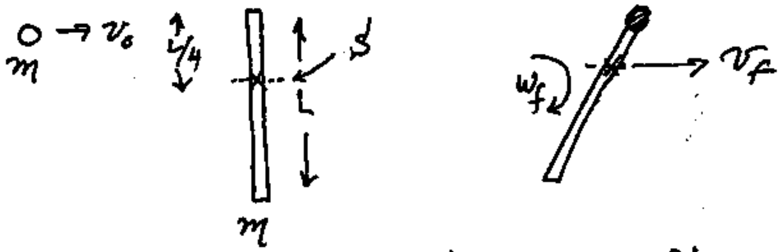
A long stick of length L and mass m lies motionless on a frictionless surface. A putty of the same mass m moves without friction on the floor with a speed of v_0 toward the stick, hits one of the ends of the stick perpendicularly and is stuck to the end.

(a) How far from the midpoint of the stick is the center of mass of the stick plus the putty at the time of the collision?

(b) What is the momentum of the stick plus putty after the collision?

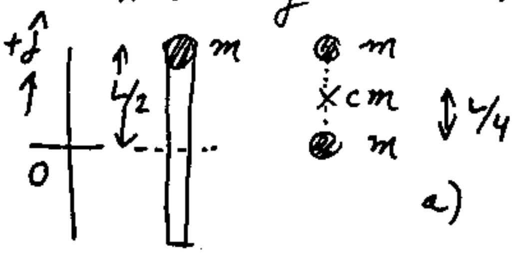
(c) What is the angular velocity of the stick plus putty after the collision?

(d) How far does the stick's center of mass move during one rotation of the stick?



b) Conservation of momentum : $m v_0 = 2m v_f \Rightarrow v_f = \frac{v_0}{2}$

Conservation of angular momentum about center of mass of stick plus putty: Point ξ in diagram.



To find center of mass, choose origin at center of stick

a) $\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = \frac{m\, L/2\, \hat{j}}{2m} = \frac{L}{4}\hat{j}$

$L_{0,\xi}$ $m\, L/4\, v_0$ initial angular momentum

$L_{f,\xi} = I^{total}_{\xi}\, \omega_f$ final angular momentum

$I^{total}_{\xi} = m(L/4)^2 + I_{stick,\xi} = \underbrace{m(L/4)^2}_{\text{moment of inertia of putty about cm of system}} + \underbrace{\left(\frac{1}{12}mL^2 + m\left(\frac{L}{4}\right)^2\right)}_{\text{parallel axis thm for moment of inertia of stick about cm of system}}$

$I^{total}_{\xi} = \frac{5}{24} m L^2$

$$L_{o,s} = L_{f,s} \Rightarrow m \frac{L}{4} v_o = \frac{5}{24} m L^2 \omega_f$$

$$\omega_f = \frac{6}{5} \frac{v_o}{L}$$

d) one rotation takes $T = \frac{2\pi}{\omega_f}$, the stick moves a distance $d = v_f T$

$$d = \left(\frac{v_o}{2}\right) \frac{2\pi}{\frac{6}{5}\frac{v_o}{L}} = \frac{5}{6} \pi L$$

Problem 6:
A mass m_1 = 1.5 kg is initially moving with a velocity v_0. It collides completely inelastically with a block of mass m_2 = 2.0 kg. The second block is attached to a spring with constant k = 5.6 x 10^3 N/m . The block and spring lie on a frictionless horizontal surface. The spring compresses a distance 2.0 x 10^{-1} m.

a) What is the velocity of the mass m_1 and the block immediately after the collision?

b) What is the initial velocity of the mass m_1 immediately before the collision?

c) If the block were attached to a very long string and hung as a pendulum, how high would the block and mass m_1 rise after the collision? Let g = 9.8 m/s^2.

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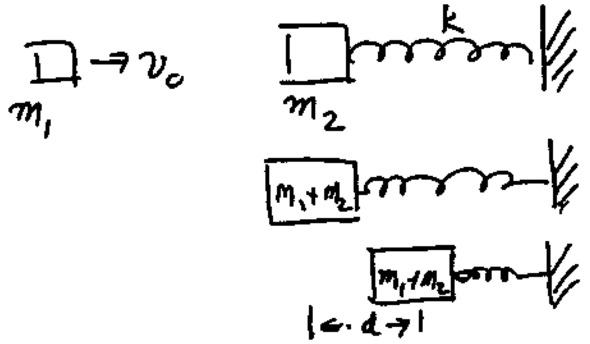


$p_0 = m_1 v_0$

$p_i = (m_1+m_2)v_i$

$\begin{cases} K_i = \frac{1}{2}(m_1+m_2)v_i^2 \\ U_i = 0 \end{cases}$

$\begin{cases} K_f = 0 \\ U_f = \frac{1}{2}kd^2 \end{cases}$

a) By conservation of momentum

$$P_0 = P_i \Rightarrow m_1 v_0 = (m_1+m_2)v_i$$

$$\Rightarrow v_0 = \frac{(m_1+m_2)v_i}{m_1}$$

By conservation of energy

$$\frac{1}{2}(m_1+m_2)v_i^2 = \frac{1}{2}kd^2 \Rightarrow v_i = \left(\frac{k}{m_1+m_2}\right)^{1/2} d$$

$$v_i = \left(\frac{5.6 \times 10^3 \text{ N/m}}{1.5 \text{ kg} + 2.0 \text{ kg}}\right)^{1/2}(2.0 \times 10^{-1} \text{ m}) = 8 \text{ m/s}$$

$$v_0 = \left(\frac{m_1+m_2}{m_1}\right)v_i = \left(\frac{1.5 \text{ kg} + 2.0 \text{ kg}}{1.5 \text{ kg}}\right)\left(\frac{8 \text{ m}}{\text{s}}\right) = 18.7 \text{ m/s}$$

c)

$$\frac{1}{2}(m_1+m_2)v_i^2 = (m_1+m_2)gh$$

$$h = \frac{v_i^2}{2g} = \frac{(8 \text{ m/s})^2}{(2)(9.8 \text{ m/s}^2)} = 3.3 \text{ m}$$

Problem 7:

A playground merry-go-round has a radius of 4.0 m and has a moment of inertia equal to 7.0×10^3 kg-m^2 with negligible friction about its vertical axis. Two children of mass 25 kg each were standing 3.0 m from the central axis on opposite sides from each other. The merry-go-round is initially at rest. A person on the ground applied a constant tangential force of 2.5×10^2 N at the rim of the merry-go-round for 1.0×10^1 seconds .

a) What was the angular acceleration of the merry-go-round?

b) What was the angular velocity of the merry-go-round when the person stopped applying the force?

c) What average power did the person put out while pushing the merry-go-round?

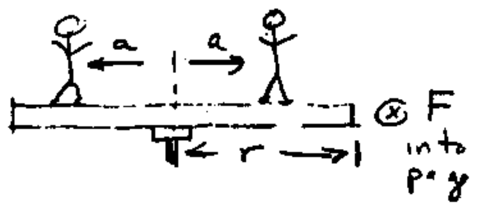
d) What was the rotational kinetic energy of the merry-go-round when the person stopped applying the force?

The two children then walked inward and stop a distance of 1.0 m from the central axis of the merry-go-round.

e) What was the angular velocity of the merry-go-round when the children reached their final position?

f) What was the change in rotational kinetic energy of the merry-go-round when the children reached their final position?

a)



$$\tau_{cm} = I^T_{cm}\,\alpha$$

$$rF = (2ma^2 + I_p)\,\alpha$$

$$I^T = (2ma^2 + I_p) = 7.450 \times 10^3 \text{ kg-m}^2$$

$$\bar{\alpha} = \frac{rF}{2ma^2 + I_p} = \frac{(4.0\text{ m})(2.5 \times 10^2\text{ N})}{((2)(25\text{ kg})(3.0\text{ m})^2 + 7.0\times10^3\text{ kg-m}^2)} = 1.3 \times 10^{-1}\ \frac{\text{rad}}{\text{sec}^2}$$

b) $\omega_1 = \bar{\alpha} t = (1.3 \times 10^{-1}\ \frac{\text{rad}}{\text{sec}})(1.0 \times 10^1\text{ sec}) = 1.3\ \frac{\text{rad}}{\text{sec}}$

c) $P = \tau\,\omega_{ave} = rF\frac{\omega_1}{2} = \frac{(4.0\text{ m})(2.5 \times 10^2\text{ N})(1.3\ \frac{\text{rad}}{\text{sec}})}{2} = 6.5 \times 10^2\text{ W}$

d) $K_1 = \frac{1}{2} I_1 \omega_1^2 \quad (\frac{1}{2})(2ma^2 + I_p)\,\omega_1^2 = 6.3 \times 10^3\text{ J}$

e) $\tau_{cm} = 0 \Rightarrow L_1 = L_2 = (2ma^2 + I_p)\,\omega_1 = (2ma_f^2 + I_p)\,\omega_2$

$$\omega_2 = \left(\frac{2ma^2 + I_p}{2ma_f^2 + I_p}\right)\omega_1 = \frac{(7450\text{ kg-m}^2)}{7050\text{ kg-m}^2}\, 1.3\ \frac{\text{rad}}{\text{sec}} = 1.4\text{ rad/sec}$$

f) $\Delta K = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_1 \omega_1^2 = 6.9 \times 10^3\text{ J} - 6.3 \times 10^3\text{ J} = .6\text{ J}$

Problem 8: *(Archimedes's Principle)*
A cylindrical beaker of mass $m_b = 1.3$ kg contains 1.5×10^3 ml of water. The beaker is placed on a scale and then a rock of mass $m_r = 2.2$ kg , suspended by a massless string, is totally immersed in the water. The water level rises by 1.5 cm. The diameter of the beaker is 0.2 m.

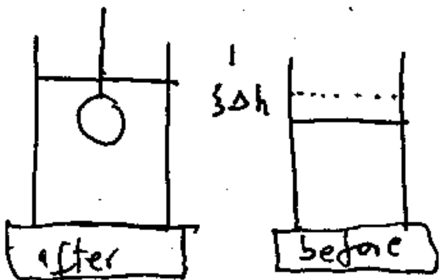
a) What mass does the scale measure before the rock is lowered into the water?

b) What mass does the scale measure after the rock is lowered into the water?

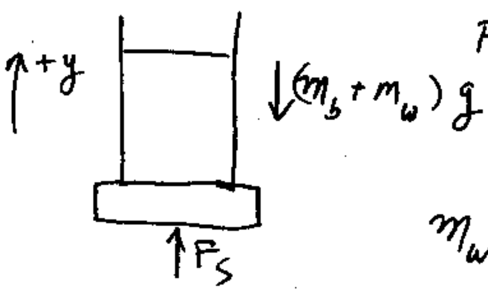
c) What is the density of the rock?

A plastic hollow sphere of the same volume as the rock has density $\rho_s = 0.6 \times 10^3\ \mathrm{kg/m^3}$. The sphere is held just below the surface of the water by a rope that is attached to the bottom of the beaker.

d) What mass does the scale now read?



Archimedes's Principle states that the buoyant force on an object has magnitude $F_b = \rho_{medium} V_{displaced}\, g$ and points upward. We shall analyze this problem by first consider the set-up before the rock is suspended



F_y	$m a_y$
$F_s - (m_b+m_w)g$	0

$$F_s = (m_b + m_w)g$$

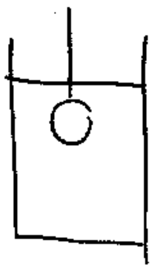
$$m_w = \rho_{H_2O} V = \left(10^3 \frac{kg}{m^3}\right)(1.5 \times 10^3\, ml)\left(\frac{cm^3}{10^0\, ml}\right)\left(\frac{1m}{10^2\, cm}\right)^3$$

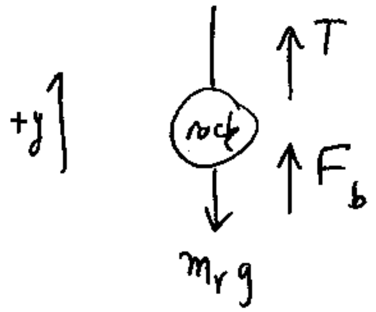
$$m_w = 1.5\ kg$$

$$F_s = (1.3\ kg + 1.5\ kg)(9.8\ m/s^2) = 27.44\ N$$

$$m_{total} = 2.8\ kg$$

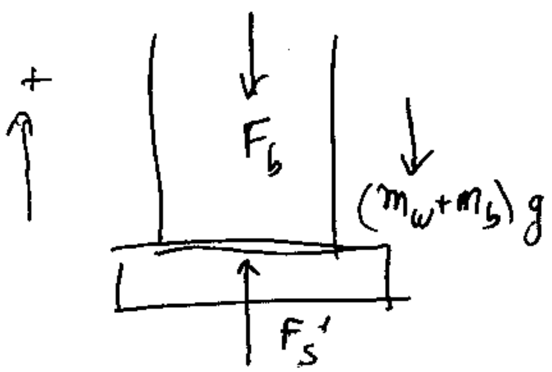
when the rock is suspended in the water, I shall consider the force diagrams on the rock, and beaker + water separately.





F_y	ma_y
$T + F_b - m_r g$	$= 0$

force diagram on rock



force diagram on water and beaker

<u>notice</u>: that there is an action-reaction force F_b acting on the water. This points down because the force of the water on the rock points up!

F_y	ma_y
$F_s' - F_b + (m_w + m_b) g$	$= 0$

$$F_s' = F_b + (m_w + m_b) g$$

the scale measures an additional
$F_b = \rho V g$

In order to calculate the displaced volume we note that the water level rose by Δh so

$$V_{displaced} = A \Delta h = ((\pi)(.1m)^2)(.015m) = 4.7 \times 10^{-4} m^3$$

$$F_b = \rho A \Delta h g$$

$$F_b = \left(10^3 \frac{kg}{m^3}\right)(\pi r^2)(.015\,m)(9.8\,m/s^2)$$

$$r = \frac{.2m}{2} = .1m \quad \text{so}$$

$$F_b = \left(10^3 \frac{kg}{m^3}\right)\left(\pi (.1m)^2\right)(.015m)\left(9.8\,m/s^2\right) = 4.6\ N$$

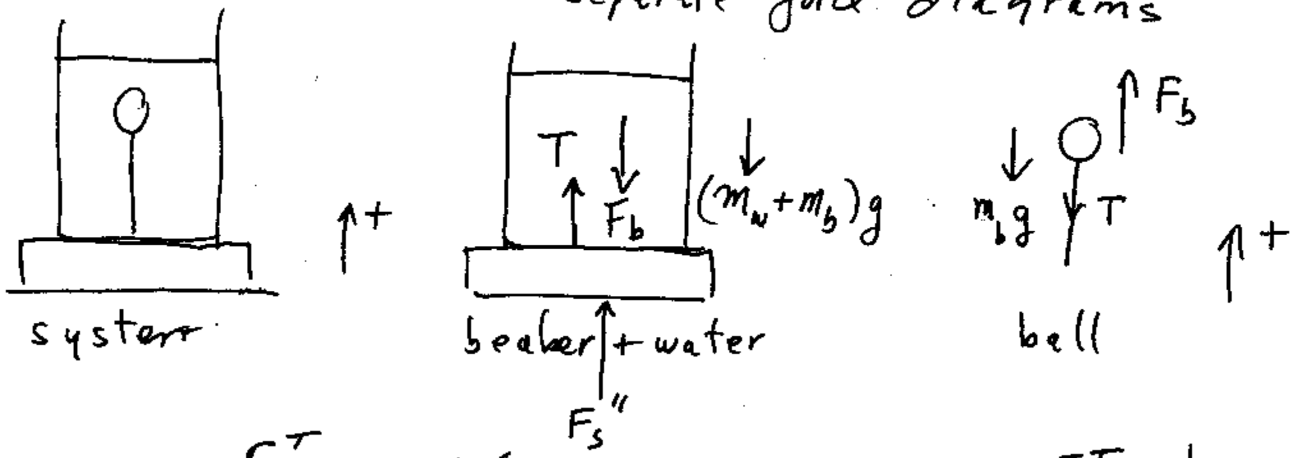
$$F_s' = 4.62N + 27.44N = 32.1\ N$$

c) the density of the rock $\rho_r = \frac{m_r}{V} = \frac{2.2\,kg}{(4.\ \times 10^{-}\ m^3)} =$

$$\rho_r = 4.7 \times 10^3 \frac{kg}{m^3}$$

Part II: Now place the plastic ball with density $\rho_b < \rho_w$. The ball will try to float.

separate force diagrams



$$\frac{F^T}{F_s'' + T - F_b - (m_w + m_b) g} \left| \frac{= (m_w + m_b)\, a}{= 0} \right. \quad (1)$$

$$\frac{F^T}{F_b - T - m_b g} \left| \frac{= m_b\, a}{= 0} \right. \quad (2)$$

Action-Reaction Pairs:

The tension T pulls the ball down so there is an action-reaction force T pulling the beaker + water up

The buoyant force F_b of the water on the ball pushes the ball up, so there is an action-reaction force F_b pushing the water + beaker down

eq (2) $\Rightarrow$ $F_b - T = + m_b g$

then eq (1) becomes $F_s'' = (m_w + m_b) g + (F_b - T)$

$F_s'' = (m_w + m_b) g + m_b g$. The scale increases by the mass of the ball

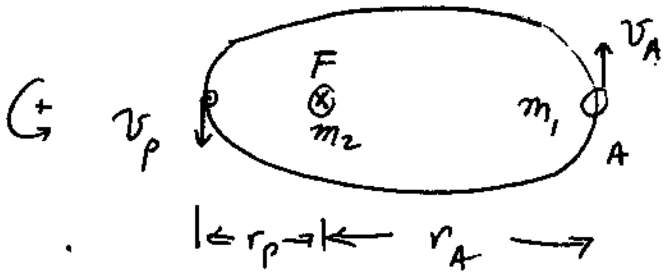
$F_s'' = 27.44\ N + (.6 \times 10^3 \frac{kg}{m^3})(4.7 \times 10^{-4} m^3)(9.8 \frac{m}{s^2}) = 30.2\ N$

where $m_b = \rho_b V = (0.6 \times 10^3\ kg/m^3)(4.7 \times 10^{-4}\ m^3) = .28\ kg$

Problem 9 *(Planetary Orbits)*
A satellite of mass m_1 is in an elliptical orbit around a planet of mass m_2 whose center is at F, one of the two foci of the ellipse ($m_2>>m_1$). P is the point of closest approach of the two masses and is a distance r_P from the foci F. The satellite's speed at the point P is v_P. The two masses are furthest apart at the point A which is a distance $r_A = 5\ r_P$.

a) What is the speed of the satellite at the point A?

b) What is the energy of the satellite?



$$r_A = 5\ r_P \qquad (0)$$

Angular Momentum is conserved about the point F: So $L_{F,A} = L_{F,P}$

$$r_A\, m_1 v_A = r_p\, m_1 v_p \implies r_A v_A = r_p v_p$$

Since $r_A = 5\, r_p \implies 5 r_p v_A = r_p v_p$

$$v_A = v_p / 5 \qquad (1)$$

Conservation of Energy:

$$E = \frac{1}{2} m_1 v_A^2 - \frac{G m_1 m_2}{r_A} = \frac{1}{2} m_1 v_p^2 - \frac{G m_1 m_2}{r_p}$$

$$\frac{1}{2} m_1 v_A^2 - \frac{G m_1 m_2}{5 r_p} = \frac{1}{2} m_1 (5 v_A)^2 - \frac{G m_1 m_2}{r_p}$$

$$G m_1 m_2 \left(\frac{1}{r_p} - \frac{1}{5 r_p} \right) = \frac{1}{2} m_1 (25 v_A^2 - v_A^2) \qquad (2)$$

We can solve eq (2) for v_A

$$v_A = \left(\frac{G m_2}{15 r_p}\right)^{1/2}$$

the energy is then

$$E = \frac{1}{2} m_1 (5 v_A)^2 - G \frac{m_1 m_2}{r_p}$$

$$= \frac{1}{2} m_1 \left(\frac{(25) G m_2}{15 r_p}\right) - G \frac{m_1 m_2}{r_p}$$

$$= -\frac{1}{6} \frac{G m_1 m_2}{r_p} < 0$$

as it should be for a bound orbit

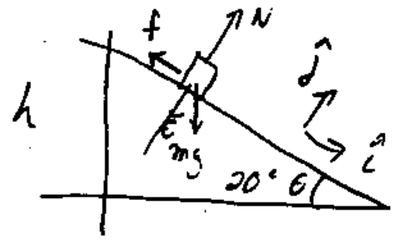
Problem 14:

A child's playground slide is 5.0 m in length and is at an angle of 2.0×10^{1} degrees with respect to the ground. A child of mass 2.0×10^{1} kg starts from rest at the top of the slide. The coefficient of sliding friction for the slide is $\mu_k = 0.2$.

a) What is the total work done by the friction force on the child?

b) What is the speed of the child at the bottom of the slide?

c) How long does the child take to slide down the ramp?



a)

$$N - mg\cos\theta = 0 \qquad (1)$$

$$mg\sin\theta - f = ma \qquad (2)$$

$$f = \mu_k N = \mu_k mg\cos\theta \qquad (3)$$

$$\Rightarrow \quad W_{friction} = -f_k d = -\mu_k mg\cos\theta\, d$$

$$= -(0.2)(20\,kg)(10\,\tfrac{m}{s^2})(\cos 20^\circ)(5.0\,m) = -1.9\times10^{2}\,J$$

b)

$$\Delta K + \Delta U = W_{friction}$$

$$(\tfrac{1}{2}mv^2 - 0) + (0 - mg\,d\sin\theta) = -f_k d$$

$$\Rightarrow \quad v = \left(\frac{2}{m}\left(mgd\sin\theta - f_k d\right)\right)^{1/2}$$

$$= \left(\frac{2}{m}\left(mgd\sin\theta - \mu_k mg\cos\theta\, d\right)\right)^{1/2} = \left(2gd(\sin\theta - \mu_k\cos\theta)\right)^{1/2}$$

$$= \left((2)(10\,\tfrac{m}{s^2})(5.0\,m)(\sin 20^\circ - .2\cos 20^\circ)\right)^{1/2} = 3.9\,m/s$$

(c) From eq (2) $\quad a = g\sin\theta - \mu_k g\cos\theta = g(\sin\theta - \mu_k\cos\theta)$

$$d = \tfrac{1}{2}at^2 \Rightarrow \quad t = \left(\frac{2d}{a}\right)^{1/2} = \left(\frac{(2d)}{(g(\sin\theta - \mu_k\cos\theta))}\right)^{1/2} = 2.5\,sec$$

Problem 11: Experiment flow

Blood flow through the vascular system of the human body is controlled by several factors. The flow is directly proportional to the pressure differential ΔP, between any two points in the system.

$$\Delta P \sim I.$$

Let L be the length of the vessel, d the diameter of the vessel, and η the viscosity. Let v(r) denote the velocity of the fluid at a distance r from the central axis. Then the laminar flow of a blood in a cylindrical vessel can be modelled by

$$v(r) = \frac{(\Delta P)(\frac{d^2}{4} - r^2)}{4\eta L}$$

where r is the distance from the center of the vessel.

There is a change of velocity with respect to distance from the center across the vessel, $\frac{dv}{dr}$. A viscous force $F_{viscous} = F_v$ acts on any cylindrical element of blood in the direction opposite the flow due to the slower moving blood outside the element. The magnitude of the viscous force is given by

$$F_v = -\eta A \frac{dv}{dr} = -\eta 2\pi r L \frac{dv}{dr},$$

where $A = 2\pi r L$ is the area of the cylindrical element.

Changes in the content of the blood can vary the viscosity. Also the arteries tend to more closely approximate the ideal model of a cylindrical pipe. Veins are more apt to change their geometry.

a) Calculate the average velocity, v_{ave}, through a vessel. What is the ratio between the average velocity and the velocity at the center $r = 0$.

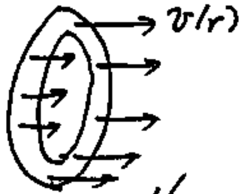
b) Calculate the total flow I through the vessel.

c) Calculate the force on the walls of the vessel (at $r = d/2$).

d) Using your result from part c), what is the net force on the vessel?

a) $$v_{ave} = \frac{1}{\pi(\frac{d}{2})^2} \int_{r=0}^{r=d/2} v(r)\, 2\pi r\, dr$$

where $2\pi r dr$ is the area of a ring of radius r and width dr.



$$V_{ave} = \frac{1}{\text{area}} \int_0^{d/2} v(r)\, 2\pi r\, dr$$

$$V_{ave} = \frac{1}{\pi\left(\frac{d}{2}\right)^2} \int_0^{d/2} \frac{\Delta P\left(\left(\frac{d}{2}\right)^2 - r^2\right)}{4\eta L}\, 2\pi r\, dr$$

$$= \frac{\Delta P\, 2\pi}{\pi\left(\frac{d}{2}\right)^2 4\eta L} \int_0^{d/2} \left(\left(\frac{d}{2}\right)^2 - r^2\right) r\, dr$$

$$= \frac{\Delta P\, 2\pi}{\pi\left(\frac{d}{2}\right)^2 4\eta L} \left(\left(\frac{d}{2}\right)^2 \frac{r^2}{2}\Big|_0^{d/2} - \frac{r^4}{4}\Big|_0^{d/2}\right)$$

$$= \frac{\Delta P}{\left(\frac{d}{2}\right)^2 2\eta L} \left(\frac{1}{2}\left(\frac{d}{2}\right)^4 - \frac{1}{4}\left(\frac{d}{2}\right)^4\right) = \frac{\Delta P}{\left(\frac{d}{2}\right)^2 2\eta L} \frac{1}{4}\left(\frac{d}{2}\right)^4$$

$$V_{ave} = \frac{\Delta P}{8\eta L}\left(\frac{d}{2}\right)^2$$

$$\frac{V_{ave}}{v(r=0)} = \frac{\dfrac{\Delta P\left(\frac{d}{2}\right)^2}{8\eta L}}{\dfrac{\Delta P\left(\frac{d}{2}\right)^2}{4\eta L}} = \frac{1}{2}$$

(b) $$I = \int_0^{d/2} v(r)\, 2\pi r\, dr = (\text{area})\, V_{ave}$$

$$I = \frac{\Delta P}{8\eta L}\left(\frac{d}{2}\right)^2 \pi\left(\frac{d}{2}\right)^2 \Rightarrow I \sim \Delta P$$

c) The force on the wall of the vessel has magnitude

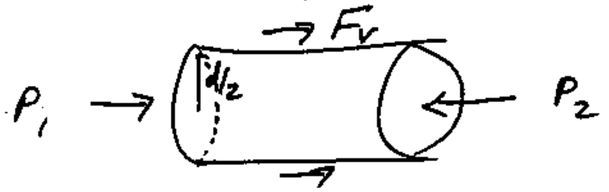
$$F_v = \left| \eta A \frac{dv}{dr} \right| = \left| \eta \, 2\pi \frac{d}{2} L \left.\frac{dv}{dr}\right|_{r=d/2} \right|$$

$$\left.\frac{dv}{dr}\right|_{r=R} = \left.\frac{\Delta P}{4\eta L}(-2r)\right|_{r=d/2} = -\frac{\Delta P}{4\eta L} 2\frac{d}{2}$$

$$F_v = \left| -\eta \, 2\pi \frac{d}{2} L \frac{\Delta P}{4\eta L} 2\frac{d}{2} \right|$$

$$F_v = \Delta P \, \pi \left(\frac{d}{2}\right)^2$$

d) The net force on the vessel is due to the viscous force and the pressure differential at the ends



$$\pi\left(\frac{d}{2}\right)^2 (P_1 - P_2) + F_v = 0$$

$$\pi\left(\frac{d}{2}\right)^2 (-\Delta P) + F_v = 0$$

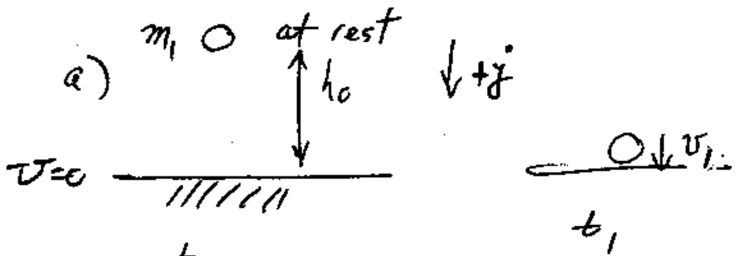
$$\pi\left(\frac{d}{2}\right)^2 \Delta P = F_v$$ agreeing with the result in part c).

Problem 7: *Impulse and Momentum*

A superball of $m_1 = 0.08$ kg, starting at rest, falls a height of 3.0 m and then collides with the ground. Assume the ball bounces up to a height of 2.0 m. Assume the collision takes place over 5 ms.

a) What is the momentum of the ball immediately before the collision?
b) What is the momentum of the ball immediately after the collision?
c) What is the average force of the table on the ball?
d) What impulse is imparted to the ball?
e) What is the change in the kinetic energy during the collision?
f) Assume that the rubber has a specific heat capacity of .48 cal/g-oC, and that all the lost mechanical energy went into heat. How much did the temperature of the ball rise as a result of the collision?

a)



$(P_0)_y = 0$ $\qquad$ $(P_1)_y = m_1 v_1$

energy is conserved

$E_0 = m_1 g h_0$

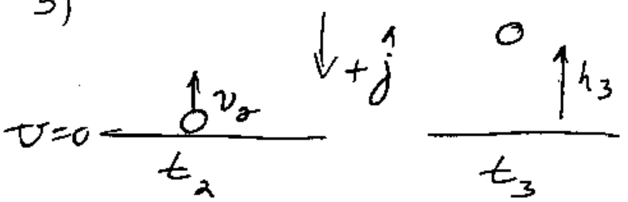
$E_1 = \frac{1}{2} m_1 v_1^2$

$E_0 = E_1$ implies

$v_1 = \sqrt{2 g h_0}$

thus $(P_1)_y = m_1 \sqrt{2 g h_0} = \left(0.08\,kg\right)\left((2)(9.8\frac{m}{s^2})(3.0\,m)\right)^{1/2} = 0.61\,kg\text{-}\frac{m}{s}$

b)



again energy is conserved implies

$v_2 = \sqrt{2 g h_3}$

$(P_2)_y = -m_1 v_2 = -m_1 \sqrt{2 g h_3} = -(0.08\,kg)\left((2)(9.8\frac{m}{s^2})(2.0\,m)\right)^{1/2} = -0.50\,\frac{kg\text{-}m}{s}$

c) $\vec{F}_{ave}\, \Delta t_{collision} = \Delta \vec{P}$

$(F_{ave})_y\, \Delta t_{collision} = (P_2)_y - (P_1)_y$

$(F_{ave})_y = \dfrac{(P_2)_y - (P_1)_y}{\Delta t} = \dfrac{-m_1\sqrt{2 g h_3} - m_1\sqrt{2 g h_0}}{\Delta t}$

$= \dfrac{(-0.50\,kg\text{-}m/s - 0.61\,kg\text{-}m/s)}{5 \times 10^{-3}\,s} = -2.22 \times 10^{2}\ N$

The $(F_{ave})_y = m_1 g - F_{\text{table on ball}}$. So

$$F_{\text{table on ball}} = m_1 g - (F_{ave})_y$$

$$F_{\text{table}} = (0.08\,kg)(9.8\,\tfrac{m}{s^2}) - (-2.22\times10^2\,N) = 2.24\times10^2\,N$$

d) $\overrightarrow{\text{impulse}} = \vec{F}_{ave}\,\Delta t = \Delta\vec{p} = -1.11\ \tfrac{kg\text{-}m}{s}\ \hat{j}$

e) $\Delta K = \frac{1}{2} m_1 v_2^{\,2} - \frac{1}{2} m_1 v_1^{\,2}$

$$= \tfrac{1}{2} m_1 (2gh_3 - 2gh_0)$$

$$\Delta K = m_1 g (h_3 - h_0) = (0.08\,kg)(9.8\,\tfrac{m}{s^2})(2.0\,m - 3.0\,m)$$

$$\Delta K = -0.78\ J$$

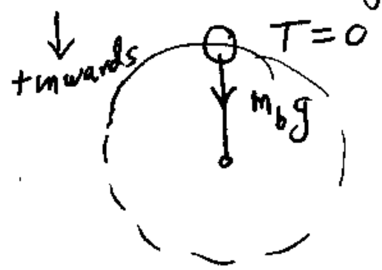
f) $\Delta Q = -\Delta K = 0.78\,J = m_1 c_1 \Delta T$

$$\Delta T = \frac{\Delta Q}{m_1 c_1} = \frac{0.78\,J}{(0.08\,kg)(.48\,\frac{kcal}{kg\text{-}{}^\circ C})(\frac{4186\,J}{kcal})} = 4.9\times10^{-3}\,{}^\circ C$$

Problem 13: *(Circular Dynamics and Elastic properties of materials)*

A mass m = 15.0 kg is attached to the end of a steel wire with an unstretched length of s_0 = 0.5 m. The mass is being whirled in a vertical circle with a constant angular velocity ω. If the tension in the wire is zero at the top of the circle, calculate the elongation of the wire when the mass is at the bottom of the circle. The cross-sectional area of the wire is A = 0.014 cm². Young modulus for steel is Y = 2.0 x 10^{11} Pa.

At the top of the orbit



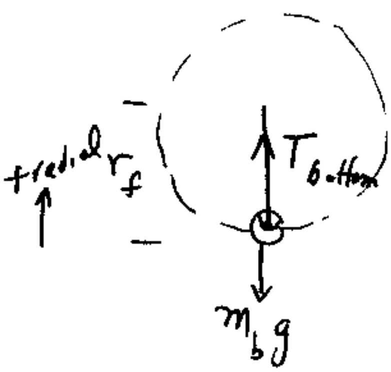
F_{radial}	$m a_r$
$m_b g$	$= m_b s_o \omega^2$

$$\Rightarrow \quad \omega = \left(\frac{g}{s_o}\right)^{1/2} = \left(\frac{9.8\,m/s^2}{.5\,m}\right)^{1/2} = 4.4\,\frac{rad}{sec}$$

At the bottom of the orbit, the steel wire has elongated to a length

$$r_f = \delta r + s_o$$

the stress on the wire is



$$\frac{T_{bottom}}{A} = Y\,\frac{\delta r}{r_o} \quad , \; Y \equiv \text{Young modulus}$$

F_{rad}	$= m_b a_{rad}$
$T_{bottom} - m_b g$	$= m_b r_f \omega^2$

use $T_b = YA\,\dfrac{\delta r}{s_o}$, $r_f = \delta r + s_o$

$$YA\,\frac{\delta r}{s_o} - m_b g = m_b(\delta r + s_o)\,\omega^2$$

solve for δr

Note: Here we could assume $r_f \simeq s_o$ and calculate T_b then calculate

$$\delta r = \frac{T_b}{AY}\,s_o$$

$$= \frac{m_b(g + s_o\omega^2)\,s_o}{AY}$$

$$\left(\frac{YA}{r_0} - m_b\,\omega^2\right)\delta r = m_b\left(g + r_0\,\omega^2\right)$$

note $r_0\omega^2 = g$

$$\delta r = \frac{2\,m_b\,g}{\frac{YA}{S_0} - m_b\,\omega^2}$$

remark $\frac{YA}{S_0} = \frac{(2\times10^{11}\,Pa)(0.014\times10^{-4}\,m^2)}{.5\,m} = 5.6\times10^{5}\,\frac{N}{m}$

$$m_b\,\omega^2 = (15\,kg)\left(4.4\,\frac{rad}{sec}\right)^2 = 2.9\times10^{2}\qquad\frac{N}{m}$$

so $\frac{YA}{S_0} >> m_b\,\omega^2$

$$\delta r = \frac{(2)(15\,kg)(9.8\,m/s^2)}{\left(\frac{(2\times10^{11}\,Pa)(1.4\times10^{-6}\,m^2)}{.5\,m} - (15\,kg)\left(4.4\,\frac{rad}{sec}\right)^2\right)} = 5.3\times10^{-4}\,m$$