Problem 1:

\[ F_y = m a_y \quad a_y = 0 \text{ since } (v_y) \text{ term is a constant!} \]

\[ F_d = mg - F_{drag} = 0 \]

\[ mg = F_{drag} \]

\[ a_v = \frac{mg}{m} = g \]

\[ v_f = \sqrt{2gL} \]

a) \[ d = \frac{v_{term} t}{2} = \frac{1.6 \text{ km}}{5.0 \times 10^4 \text{ m/s}} \]

b) \[ W_{grav} = mgd = \left(8.0 \times 10^4 \text{ kg}\right) 9.8 \left(1.6 \times 10^3 \text{ m}\right) = 1.25 \times 10^8 \text{ J} \]

c) \[ W_{drag} = -F_{drag}d = -mgd = -1.25 \times 10^6 \text{ J} \]

d) \[ P_{drag} = \frac{W_{drag}}{t} = \frac{-1.25 \times 10^6}{3.2 \times 10^6} = -3.92 \times 10^4 \text{ W} \]

e) \[ E_{drag} = W_{drag} = -mgd = -3.92 \times 10^4 \text{ J} \]. These energy shows up as heat; heating both skydiver and the air.

Problem 2:

\[ F_{rock} = m a_{rock} \]

\[ (1) \quad T + mg = \frac{m v_f^2}{L} \]
a) 

Mod. Energy is conserved since $T$ is always perpendicular to the displacement

\[ \Delta K + \Delta \text{P.E.} = W_{n.c.} \]

\[ \frac{1}{2} m (v_f^2 - v_0^2) + mg(\Delta l) = 0 \]

Solve for \( \frac{v_f^2}{f} = v_0^2 = 2g(\Delta l) \)

\[ v_f = \left( \frac{7.0 \text{ m/s}^2 - (2)(9.8 \text{ m/s}^2)(2)(1.5 \text{ m})}{2} \right)^{\frac{1}{2}} = 5.4 \text{ m/s} \]

b) \[ T = \frac{m v_f^2}{l} - mg = \frac{m}{l} (v_0^2 - 2g(\Delta l)) - mg \]

\[ T = \frac{m v_0^2 - 5 \text{ m} g}{l} = \frac{(1.1 \text{ kg})(7.0 \text{ m/s})^2 - (5)(1.1 \text{ kg})(9.8 \text{ m})}{(1.5 \text{ m})} \]

\[ T = 4.9 \text{ N} \]

Problem 3: the pen should have zero velocity at \( x = \infty \)

a) \( x_{f0} \quad x_{f} = 0 \quad v_f = 0 \)

Energy is conserved

\[ \Delta K + \Delta \text{P.E.} = W_{n.c} \]

\[ 0 + \frac{1}{2} k (x_f^2 - x_0^2) = \Delta E_m + \frac{1}{2} \frac{m_1}{m_2} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\[ -\frac{1}{2} k x_0^2 + 6 m_1 m_2 = 0 \rightarrow x_0 = \frac{2 m_1 m_2}{R_p k} \]

\[ x_0 = \left( \frac{(2)(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(0.01 \text{ kg})(2.6 \times 10^{-3} \text{ kg})}{5.0 \times 10^3 \text{ m}} \right)^{\frac{1}{2}} = 0.42 \text{ m} = 4.2 \text{ cm} \]
\[ r_s^3 = \frac{Gm_p M}{4\pi^2} \]

\[ r_s = 6.1 \times 10^3 \text{ m} \]

\[ v_s = \frac{2\pi r_s}{T} = \frac{(2\pi)(6.1 \times 10^3 \text{ m})}{7.2 \times 10^3 \text{ sec}} = 5.3 \text{ m/s} \]

d) At the radius \( r_s \), the conservation of mechanical energy can be used to calculate the velocity of the pen.

\[ \Delta k + \Delta E = -e_{nc} \]

\[ \frac{1}{2} m v_1^2 - \frac{1}{2} k x_0^2 - Gm_1 m_2 \left( \frac{1 - \frac{1}{r_s^2}}{r_p} \right) = 0 \quad \text{(1)} \]

So eq (1) becomes

\[ \frac{1}{2} \left( \frac{G m_1}{r_s} \right) v_1^2 - \frac{6.1 \times 10^3 \text{ m}^2}{r_p} \cdot \frac{r_s^2}{r_p} = 0 \]

\[ v_1 = \frac{(\frac{2G m_1}{r_s}) \cdot v_2}{(2)(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(2.6 \times 10^{15} \text{ kg})} \]

\[ \frac{r_s}{6.1 \times 10^3 \text{ m}} \]

\[ v_1 = 7.54 \text{ m/s} \]
Problem 4
\[ v_f = \frac{\Delta k + \text{OPE}}{W_{nc} - \Delta h} \]
\[ \frac{1}{2}mv_f^2 + mg(\Delta h) = F_{floor} \Delta h \]
\[ F_{floor} = 3 \text{ mg} \]
Solve for \( v_f \):
\[ v_f = \sqrt{2\left(\frac{F_{floor} - mg}{m}\right) \Delta h} = \sqrt{\frac{(2)(2mg)\Delta h}{m}} \]
\[ v_f = \sqrt{2g\Delta h} = \sqrt{2\left(\frac{9.8 \text{ m/s}^2}{0.2 \text{ m}}\right)\frac{1}{2}} = \frac{2.8 \text{ m}}{s} \]

Problem 5
a) \[ \Delta Q = mc_w \Delta T \]
\[ = (2.5 \text{ kg})(4190 \text{ J/kg K})(28 \text{ K}) = 2.9 \times 10^4 \text{ J} \]
b) \[ P_{ave} = \frac{\Delta Q}{\Delta t} = \frac{2.9 \times 10^4 \text{ J}}{60 \text{ s}} = 4.9 \times 10^2 \text{ W} \]

Problem 6:
\[ v \rightarrow \begin{array}{l} t_0 \rightarrow \text{ m} \rightarrow \text{ v} \rightarrow \theta + \text{ c} \\
\end{array} \]
\[ t' \]
\[ v_1 \leftarrow \begin{array}{l} m_1 \rightarrow \text{ m} \rightarrow \text{ v}_1 \rightarrow \text{ m}_2 \rightarrow \text{ v}_2 \\
\end{array} \]

Momentum is conserved because there are no external forces. Energy is not conserved due to the explosion.
\[ \Delta P_x = 0 \]
\[ m_2v_2 - m_1v_1 = m v = 0 \]
\[ v_2 = \frac{m v + m_1v_1}{m_2} = \frac{(2.0 \text{ kg})(2.0 \text{ m/s}) + (0.5 \text{ kg})(1.0 \text{ m/s})}{1.5 \text{ kg}} \]
\[ = 6 \text{ m/s} \]
\[ \Delta K_f = \frac{1}{2} m_1 v_{1,f}^2 - \frac{1}{2} m_2 v_{2,f}^2 = \text{work} \]

\[(\frac{1}{2})(1.5\text{kg})(6\text{m/s})^2 + (\frac{1}{2})(1.5\text{kg})(10\text{m/s})^2 - \frac{1}{2}(2.0\text{kg})(2.0\text{m/s})^2 = 48 \text{J} \]

is the increase in kinetic energy due to the explosion.

**Problem 7:**

\[ \begin{align*}
& \frac{v_{1,f}}{v_{2,f}} = 1 \\
& \frac{m_1}{m_2} = 0 \rightarrow v_{1,f} = v_{2,f} \\
& m_1 v_{1,0} - m_2 v_{2,0} = m_2 v_{2,f} \cos \theta_2,f \\
& \Delta \theta = 0 \Rightarrow \theta_{1,0} = \theta_{2,f} \\
& 0 = m_1 v_{1,f} - m_2 v_{2,f} \sin \theta_2,f \\
& \Delta K = 0 \Rightarrow K_0 = K_f \\
& \frac{1}{2} m_1 v_{1,0}^2 + \frac{1}{2} m_2 v_{2,0}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2 \tag{3}
\end{align*} \]

Additionally, we are told that \( v_{1,f} = \frac{v_{1,0}}{2} \)

Eq (2) can be rewritten using this fact as

\[ m_2 v_{2,f} \sin \theta_2,f = \frac{m_1 v_{1,0}}{2} \tag{4} \]

\[ m_2 v_{2,f} \cos \theta_2,f = m_1 v_{1,0} - m_2 v_{2,0} \tag{5} \]

So dividing these equations yields

\[ \tan \theta_{2,f} = \frac{m_1 v_{1,0}}{2}{m_1 v_{1,0} - m_2 v_{2,0}} \]
\[
\sin \theta \tan \Theta_{2f} = \tan 45^\circ = 1 \quad \text{we have}
\]

\[
1 = \frac{m_1 \, v_{1\text{o}}}{m_1 \, v_{1\text{o}} - m_2 \, v_{2\text{o}}} \quad \text{or} \quad m_1 \, v_{1\text{o}} - m_2 \, v_{2\text{o}} = \frac{1}{2} \quad m_1 \, v_{1\text{o}}
\]

which we can solve for \( v_{2\text{o}} \)

\[
v_{2\text{o}} = \frac{1}{2} \quad \frac{m_1}{m_2} \quad v_{1\text{o}}
\]

\[
v_{2\text{,}f} = \frac{m_1}{m_2} \quad \frac{v_{1\text{o}}}{2} \quad \sin \theta_2 \cdot f = \frac{m_1 \, v_{1\text{o}}}{m_2} \quad \frac{m_1 \, v_{1\text{o}}}{m_2} \quad \frac{v_{1\text{o}}}{2}
\]

So \( \alpha (1) \) can also be solved for \( v_{2\text{,}f} \)

\[
\frac{1}{2} \quad m_1 \, v_{1\text{o}}^2 + \frac{1}{2} \quad m_2 \left( \frac{1}{2} \quad \frac{m_1}{m_2} \right) \quad v_{1\text{o}}^2 = \frac{1}{2} \quad m_1 \, \left( \frac{v_{1\text{o}}}{2} \right)^2 + \frac{1}{2} \quad m_2 \, \left( \frac{m_1 \, v_{1\text{o}}}{m_2} \right)^2
\]

or

\[
\frac{1}{2} \quad m_1 \, v_{1\text{o}}^2 + \frac{1}{2} \quad m_2 \quad \frac{m_1^2}{4 \quad m_2^2} \quad v_{1\text{o}}^2 = \frac{1}{2} \quad m_1 \, \left( \frac{v_{1\text{o}}}{2} \right)^2 + \frac{1}{2} \quad m_2 \quad \frac{m_1^2}{m_2^2} \quad \frac{v_{1\text{o}}^2}{2}
\]

\[
\frac{3}{4} \quad m_1 \, v_{1\text{o}}^2 = \frac{1}{4} \quad \frac{m_1 \, v_{1\text{o}}^2}{m_2}
\]

\[
\Rightarrow \quad 3 = \frac{m_1}{m_2}
\]
Problem 8:

The equilibrium position is already a slightly stretched position since at equilibrium:

\[ F_y = m g \]

\[ m g - k y_{eq} = 0 \]

\[ y_{eq} = \frac{m g}{k} \]

Then when the system is stretched an additional distance \( y_0 \) at \( t = 0 \):

\[ F_y = m d^2 y \]

\[ m g - k (y + y_0) = m d^2 y \]

\[ \frac{m g - k y_{eq}}{m} - k y = m d^2 y \]

\[ y_0 = \frac{m d^2 y}{d t^2} \]

\[ m d^2 y + k y = 0 \]

Simple harmonic motion about equilibrium position:

\[ y = A \cos \sqrt{\frac{k}{m}} t + B \sin \sqrt{\frac{k}{m}} t \]

\[ A = y_0, \quad B = \frac{y_0}{\sqrt{\frac{k}{m}}} = 0 \] released from rest

\[ \frac{d y}{d t} = -\sqrt{\frac{k}{m}} A \sin \sqrt{\frac{k}{m}} t + \sqrt{\frac{k}{m}} B \cos \sqrt{\frac{k}{m}} t \]

Period:

\[ T = 2 \pi \sqrt{\frac{m}{k}} \]

\[ T^2 = 4 \pi^2 \frac{m}{k} \]
we can find the velocity using

\[ y = y_0 \cos \sqrt{\frac{k}{m_1}} t, \]
\[ v_y = -\sqrt{\frac{k}{m_1}} y_0 \sin \sqrt{\frac{k}{m_1}} t \]

noting that when \( \sqrt{\frac{k}{m_1}} t = \frac{\pi}{2} \), \( \cos(\sqrt{\frac{k}{m_1}} t) = 0 \)
so \( y = 0 \), mass is back at orig. pos.
Also, \( \sin \left( \frac{\pi}{2} \right) = 1 \) so

\[ v_y = -\sqrt{\frac{k}{m_1}} y_0 \] at orig. pos. \( t_{eq} = \frac{\pi}{2} \sqrt{\frac{m_1}{k}} \)

c) since a new mass \( m_2 = 2m_1 \), \( m_{total} = 3m_1 \), and \( T = 2\pi \sqrt{\frac{3m_1}{K}} \)

d) since the collision occurred when the mass was completely compressed, the velocity was zero, hence for the collision \( \Delta K = 0 \), no energy was lost. Therefore, the new system of mass \( 3m_1 \) will satisfy

a new equilibrium condition

\[ 3m_1 g = k y_{eq} \Rightarrow y_{eq} = \frac{3m_1 g}{k} \]
and the oscillations are about this position.
So the new equilibrium position is lowered by \( \frac{2m_g}{k} \).

When the rubber bands were fully compressed by \( y_0 \), the collision occurred. The

So with respect to the new equilibrium position, the stretch is

\[
y_0 + \frac{2m_g}{k}
\]

Hence, when the system is fully stretched, the mass \( 3m_1 \) is at a position

\[
y_0 + \frac{4m_g}{k}
\]

from the original equilibrium position.