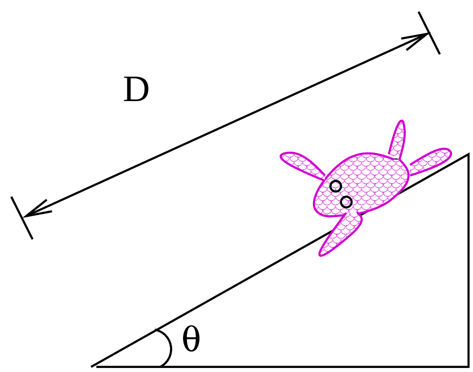
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X — Fall Term 2001

EXAM 2 Solutions

Problem 1:



The Sparkly Frog slides down a ramp angled at $\theta = 30^\circ$ to the floor. The ramp is $D = 1.5$ m long. The coefficient of kinetic friction is $\mu_k = 0.20$.

a. (15 points) If the Frog is released from rest at the top of the ramp, how much time does it take it to reach the bottom?

b. (15 points) Suppose somebody puts a gluey substance on the ramp to make it less slippery. What is the smallest coefficient of static friction μ_s such that the Frog will not slide down the ramp?

Solution 1:

(a) Set up a coordinate system so that y is normal to the slide and x is along the slide as shown.

$$\Sigma F_y = N - mg\cos\theta$$

$$\Sigma F_y = 0 \Longrightarrow N = mg\cos\theta$$

$$\Sigma F_x = mg\sin\theta - f = mg\sin\theta - mg\cos\theta\mu_k$$

Apply Newton's second law

$$\Sigma F_x = ma_x$$

so

$$ma_x = mg\sin\theta - mg\cos\theta\mu_k$$

$$a_x = g(\sin\theta - \cos\theta\mu_k) = 3.2m/s^2$$

$$D = \frac{1}{2}at^2 \Longrightarrow t = \sqrt{\frac{2D}{a}} = 0.97sec$$

(b) Again

$$\Sigma F_y = 0 \Longrightarrow N = mg\cos\theta$$

$$\Sigma F_x = mg\sin\theta - f_s = 0$$

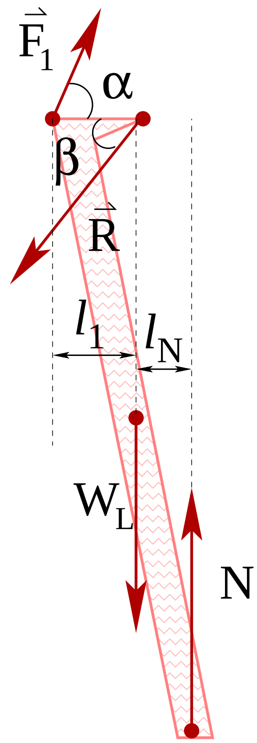
$$\Longrightarrow f_s = mg\sin\theta$$

Now $0 \leq f_s \leq \mu_s N, 0 \leq f_s \leq mg\cos\theta\mu_s$

$$\Longrightarrow mg\cos\theta\mu_s^{min} = mg\sin\theta$$

$$\mu_s^{min} = \tan\theta = 0.58$$

Problem 2:



Consider an elderly person with a hip injury. As we discussed in class, the force of the hip socket R on the head of the femur can be quite large, approximately 2.5 times the weight of the body W, possibly causing considerable pain. The use of a cane can reduce the force and hence the pain considerably. When a cane is used, the normal force acting on the foot is reduced to 5/6 of W; furthermore the horizontal distance l_N between where the hip socket force acts and where the normal force on the foot acts is reduced. The diagram shows the forces acting on the leg when a cane is in use:

- $W_L = W/7$ is the weight of the leg. It acts at the center of mass of the leg, directly below the point where the force $\vec{R}$ acts.
- $N = 5W/6$ is the normal force from the ground.

- $\vec{R}$ is the force of the socket on the head of the femur, which acts at an angle β with the horizontal.
- $\vec{F_1}$ is the force of the hip abductor muscle, which acts at an angle $\alpha = 70°$ with the horizontal.
- $l_1 = 2.75$ inches is the horizontal distance between the points where the forces $\vec{R}$ and $\vec{F_1}$ act.
- $l_N = 1.85$ inches is the horizontal distance between the points where the forces $\vec{R}$ and $\vec{N}$ act.

a. (20 points) Find an expression for the magnitude of $\vec{F_1}$ in terms of W.

b. (20 points) Find an expression for the magnitude of the force $\vec{R}$ on the head of the femur, in terms of W. Compare this to the case where no cane is used.

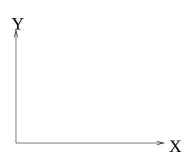
Solution2:

(a) Use the head of the femur as the pivot point and apply $\Delta\vec{\tau} = 0$ on the leg

$$Nl_N - F_1 l_1 \sin 70^0 = 0$$

$$\Longrightarrow F_1 = \frac{Nl_N}{l_1 \sin 70^0} = 0.6W$$

(b) Set up a coordinate system as shown below.



The magnitude of the force $\vec{R}$ is

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\Sigma F_x = F_1 \cos 70^0 - R\cos\beta = 0 \Longrightarrow R\cos\beta = R_x = F_1 \cos 70^0$$

$$\Sigma F_y = N - W_L + F_1 \sin 70^0 - R\sin\beta = 0$$

$$\Longrightarrow R\sin\beta = 5W/6 - W/7 + 0.6W\sin 70^0 = 1.25W = Ry$$

Thus,

$$R = \sqrt{R_x^2 + R_y^2} = 1.27W$$

When there is no cane used, the force R on the head of the femur can be quite large, approximately 2.5 times the weight of the body W. With care, the force R is reduced by a factor of 2, which significantly reduces the pain.

Problem 3:

In experiment CF, the unstretched rubber band has a length l_0, and the nut attached at the end of it has mass m. When the rubber band is stretched, the restoring force is proportional to the stretch with constant k. The rubber band is rotated with a constant frequency f in a circular orbit. A student looking down at the setup sees a counterclockwise orbit.

a. (8 points) Draw the stationary patterns you would expect on your Experiment CF motor for rotational frequencies of 10 Hz, 12 Hz, 15 Hz and 20 Hz.

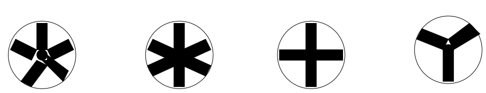
b. (9 points) Derive an expression for the orbital radius of the nut in terms of k, l_0, f and m.

c. (5 points) Suddenly the rubber band breaks. Draw a sketch of the motion of the nut after the break, and give a brief explanation of your sketch.

d. (8 points) Suppose the motor was turning at a frequency $f = 30$ Hz before the break and radius of the circular path was 150 mm. $m = 1.0 \times 10^{-4}$ kg. What was the magnitude and direction of the tension in the rubber band just before the break?

Solution 3:

(a) Stationary patterns:



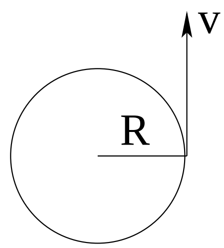
12 Hz 10 Hz 15 Hz 20 Hz

(b) Let the orbital radius of the nut be R, the spring force is $F = -k(R - l_0)$, which is the centripetal force for the uniform circular motion of the nut. Thus

$$k(R - l_0) = m\omega^2 R = m(\frac{2\pi}{T})^2 R = m4\pi^2 f^2 R$$

$$\implies R = \frac{kl_0}{k - m4\pi^2 f^2}$$

(c) The nut is in a uniform circular motion in the horizontal plane before the rubber band breaks and its position is shown in the sketch at the moment just before the break.



After the rubber brand broke, the nut will move with velocity along the tangential direction with magnitude

$$v = \omega R = 2\pi f R$$

and maintain this velocity in the horizontal plane assuming the surface is smooth and friction is zero.

(d)

$$a_c = \omega^2 R = 4\pi^2 f^2 R = 4\pi^2 (30)^2 0.15 = 5324.2 m/s^2$$

$$fc = ma_c = 0.53N$$

The magnitude of the tension is 0.53 N and it's toward the center of the circular trajectory along the radical direction (inward).