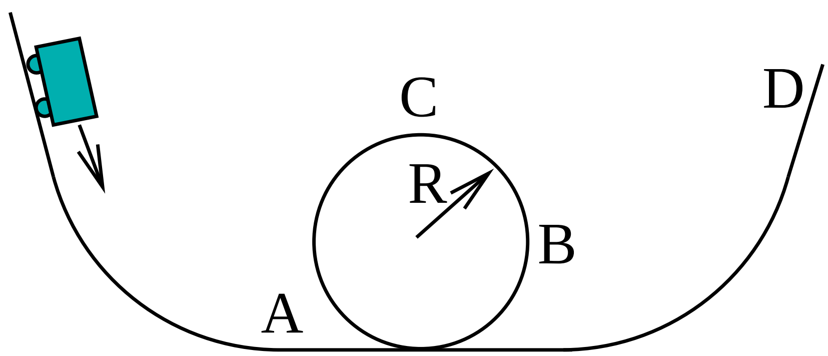
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X Fall Term 2001

Solutions to EXAM 3

Problem 1:

After its release at the top of the first rise, a roller coaster car moves with negligible friction. The roller coaster has a circular loop of radius $R = 20.0$ m. The car barely makes it around the loop: at the top of the loop, the riders are upside-down and feel weightless.



a. (8 points) Find the speed of the roller coaster at the top of the loop (position C).

b. (7 points) Find the speed of the roller coaster at position A, at the bottom.

c. (7 points) Find the speed of the roller coaster at position B, halfway up the loop.

d. (8 points) Find the difference in height between positions A and D if the speed at D is 10.0 m/s.

Solution 1:

(a) At position C,

$$mg = m\frac{v^2}{R} \Longrightarrow v = \sqrt{gR} = 14 \text{ m/s}$$

(b) Let's choose position A as the zero gravitational potential energy position. From energy conservation

$$mg2R + \frac{1}{2}mv^2 = \frac{1}{2}mv_A^2$$

$$\Longrightarrow v_A = \sqrt{5gR} = 31.3 \text{ m/s}$$

(c) At position B,

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgR$$

$$\Longrightarrow v_B = \sqrt{3gR} = 24.2 \text{ m/s}$$

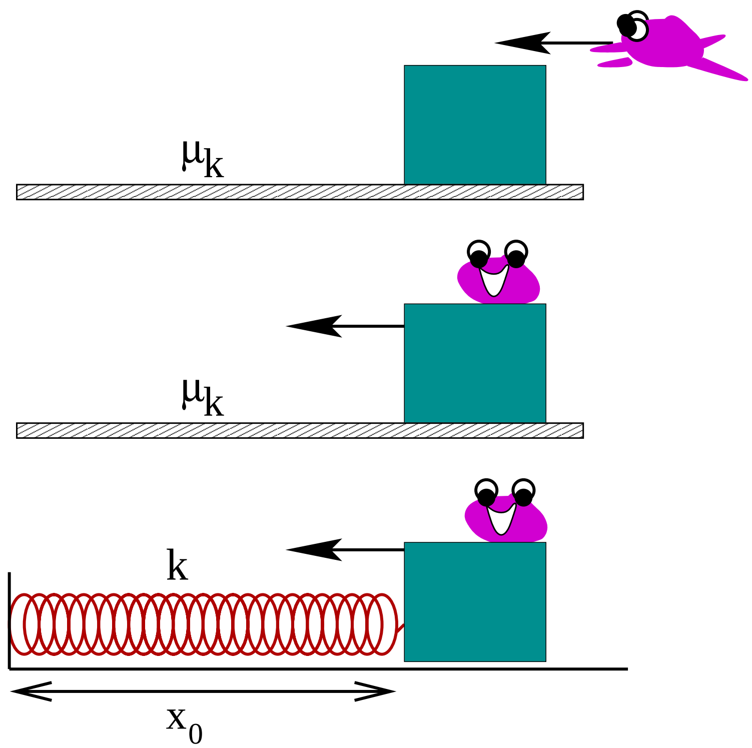
(d) $v_D = 10.0$ m/s

$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_D^2 + mgh$$

$$\Longrightarrow 5gR - v_D^2 = 2gh \Longrightarrow h = 45.0 \text{ m}$$

Problem 2:

Glorgs are small frog-like creatures which inhabit the planet Mongo, where the acceleration of gravity is 46 m/s^2. On Mongo, a 20 g glorg launches itself horizontally towards a 150 g block at rest on a horizontal surface. Glorgs are rather sticky and this one sticks to the block when it hits, immediately after which the glorg and block start to slide. The coefficient of friction between the block and the surface is $\mu_k = 0.65$. The block and glorg slide for 20 cm before coming to rest.



a. (11 points) What was the speed of the glorg before impact?

b. (11 points) Suppose that now the block is attached to a spring of constant $k = 100$ N/m at equilibrium, and this time the surface is frictionless. The glorg arrives at the block with the same velocity as before. How far does the spring compress after the glorg hits it?

c. (12 points) Find an expression for the position as a function of time of the block in the spring case, if the pre-impact position of the block is x_0.

d. (6) If the glorg were brought to Earth, would your answers to parts b and c change? Explain. Assume that the masses, coefficient of friction, spring constant, and glorg velocity remain unchanged.

solution 2:

Set up a one-dimensional coordinate so that positive x points right.

(a) Assuming the speed of the glorg before the impact is v_g, after the impact, the speed of the glorg and the block is v', from momentum conservation

$$m_g v_g = (m_g + m_b)v'$$

$$\Longrightarrow v' = \frac{m_g v_g}{m_g + m_b} = \frac{2}{17}v_g$$

Once the block and the glorg start to slide, the acceleration is

$$a = \mu_k g = 29.9 \text{ m/s}^2$$

and the direction is towards right.

$$v^2 = v_0^2 + 2ax \Longrightarrow 0 = v_0^2 - 2a \times 0.2$$

$$v_0 = v' \Longrightarrow v' = 3.46 \text{ m/s}^2$$

Thus

$$v_g = 8.5v' = 29.4 \text{ m/s}$$

(b) Surface is frictionless, so from energy conservation

$$\frac{1}{2}(m_g + m_b)v'^2 = \frac{1}{2}kx^2$$

$$\Longrightarrow x = 14.3 \text{ cm}$$

(c) Let $x = x' - x_0$, from $F = -kx$,

$$(m_g + m_b)\frac{d^2x}{dt^2} + kx = 0$$

So $x = A\cos(\omega t + \phi)$ and $v = -A\omega\sin(\omega t + \phi)$, with

$$\omega = \sqrt{\frac{k}{m_g + m_b}} = 24.25/s$$

Initial condition

$$t = 0, x' = x_0 \Longrightarrow \phi = \frac{\pi}{2}$$

$$t = 0, v = -v' \Longrightarrow A = \frac{v'}{\omega} = 0.143 \text{ m}$$

So

$$x' - x_0 = 0.143\cos(24.25t + 90^\circ)$$

$$\Longrightarrow x' = x_0 + 0.143\cos(24.25t + 90^\circ)$$

(d) If g = 9.8 m/s^2, the answer to part (b) remains the same, because it does not depend on the gravity. For part (c), the expression would be the same since it does not depend on g either.

Problem 3:

In Experiment ET, you measure the following calibration data:

Thermistor resistance (Ω)	Temperature ($^\circ$C)
122.7	20.3
86.4	30.2
73.5	35.1
64.5	39.3
57.5	43.0

Next, you measure the following heating data:

Thermistor resistance (Ω)	Temperature ($^\circ$C)	Time (seconds)
118.0		0
111.7		105.
105.3		210.

The mass of water in your cup is 66.1 grams and you have your LVPS set to 10 V. You use 15 cm of 800 wire, so that its resistance is 0.15 Ω, and you measure a voltage 0.074 V across it. The current in the wire is given by $I = V_{\text{wire}}/R$. The power delivered to the bulb is $P_{\text{bulb}} = IV_{\text{bulb}}$, and $V_{\text{bulb}} = V_{\text{LVPS}} - V_{\text{wire}}$.

To help answer the following questions, graph paper is provided if needed.

a. (10 points) Fill in the center column of the table, i.e. calculate the temperatures corresponding to the measured resistances.

b. (10 points) What is the power absorbed by the water?

c. (12 points) Calculate the specific heat of water from your data.

d. (8 points) Suppose your clumsy lab partner accidentally dropped a 5 gram chunk of lead into your styrofoam cup before you started the experiment. The specific heat of lead is 0.13 J/(g deg). Estimate how much this would affect your results.

solution 3:

$$R = R_0 e^{-\alpha T}$$

$$\Longrightarrow \alpha = \frac{lnR_2 - lnR_1}{T_1 - T_2} = \frac{ln122.7 - ln57.5}{43 - 20.3} = 0.0334$$

Since $R = R_0 e^{-0.0334T}$, T in ^{0}C. From R=122.7 Ω, T=20.3 ^{0}C , one gets $\rightarrow R_0 = 241.7\Omega$

(a)

$$R_1 = 118.0 \Longrightarrow T_1 = 21.467\ ^0\text{C}$$

$$R_2 = 111.7 \Longrightarrow T_1 = 23.113\ ^0\text{C}$$

$$R_3 = 105.3 \Longrightarrow T_1 = 24.877\ ^0\text{C}$$

(b) Power = IV = $\frac{V_{wire}}{R_{wire}}(V_{LVPS} - V_{wire}) = 4.897$ watts

(c)

$$\Delta Q = cm\Delta T \Longrightarrow c = \frac{\Delta Q}{m\Delta T} = 4.56 \text{ J/(g deg)}$$

(d)

$$\frac{\Delta Q_{lead}}{\Delta Q_{water}} = \frac{c_1 m_1 \Delta T}{c_2 m_2 \Delta T} = \frac{0.13 \times 5}{4.56 \times 66.1} = 0.002$$

Thus the effect is negligible.