Massachusetts Institute of Technology
Physics Department

8.01X **Fall 2001**

Handed out: Nov 30 **Due: Dec 7 at 5 pm in 4-339B**

Problem 1: EXPERIMENT FL: Flow ANALYSIS

In Experiment Flow, you made a plot of water level height h vs. time t. Here's how to interpret your data in terms of the coefficient of viscosity. The volume flow rate of water in your bottle is given by

$$\frac{V_f}{t} = -A\frac{dh}{dt} \tag{1}$$

where A is the cross sectional area of your water bottle. According to Poiseuille's Equation, the volume flow rate from the pipe is given by

$$\frac{V_f}{t} = \frac{\pi(P_1 - P_2)R^4}{8\eta L} \tag{2}$$

where L is the length of the pipe, R is the radius of the pipe, η is the coefficient of viscosity, $P_2 = P_{atm}$, is the atmospheric pressure at the open end of the pipe, and P_1 is the pressure at the opening of the pipe inside the bottle. The pressure P_1 is given by Pascal's Law

$$P_1 = P_{atm} + \rho g h \tag{3}$$

where $\rho = 1.0 \times 10^3 \frac{kg}{m^3}$ is the density of water, $g = 9.8\frac{m}{s^2}$, and h is the height of the water level above the hole. Thus $P_1 - P_2 = \rho g h$.

a) Setting eq.'s (1) and (2) equal yields a differential equation for the rate of change of the height of the water as a function of time

$$\frac{dh}{dt} = -\frac{\pi(P_1 - P_2)R^4}{8\eta LA} \tag{4}$$

Using Pascal's Law (3) for the pressure difference in equation (4) then gives

$$\frac{dh}{dt} = -\frac{\pi\rho g h R^4}{8\eta LA}.$$

This is a first order linear differential equation for the height and can be integrated by the techniques of separation of variables:

$$\int_{h_0}^{h(t)} \frac{dh}{h} = \int_0^t -\frac{\pi\rho g R^4}{A8\eta L}dt$$

Performing the integration of equation, we have

$$\ln h(t) - \ln h_0 = -\frac{\pi\rho g R^4}{A8\eta L}t \tag{5}$$

Now exponentiate both sides of eq. (5) to derive the height as a function of time:

$$h(t) = h_0 \exp\left(-\frac{\pi \rho g R^4}{A 8 \eta L} t\right). \tag{6}$$

By making a semi-log plot of the height vs. average time, the coefficient of viscosity can be calculated from the slope of the best-fit straight line according to

$$\eta = -\frac{\pi \rho g R^4}{A 8 L (\text{slope})}. \tag{7}$$

I. Calculating the Viscosity of Water:

Using eq. (7), calculate your experimental values for the viscosity of water for each length of pipe **using the data you obtained in Experiment Flow?** What was the temperature of the water for your measurements? The coefficient of viscosity varies as a function of the temperature of the water.