Problem 1: EXPERIMENT FL: Flow ANALYSIS

In Experiment Flow, you made a plot of water level height \( h \) vs. time \( t \). Here's how to interpret your data in terms of the coefficient of viscosity. The volume flow rate of water in your bottle is given by

\[
\frac{V_t}{t} = -A \frac{dh}{dt}
\]  (1)

where \( A \) is the cross sectional area of your water bottle. According to Poiseuille's Equation, the volume flow rate from the pipe is given by

\[
\frac{V_t}{t} = \frac{\pi (P_1 - P_2) R^4}{8 \eta L}
\]  (2)

where \( L \) is the length of the pipe, \( R \) is the radius of the pipe, \( \eta \) is the coefficient of viscosity, \( P_2 = P_{\text{atm}} \), is the atmospheric pressure at the open end of the pipe, and \( P_1 \) is the pressure at the opening of the pipe inside the bottle. The pressure \( P_1 \) is given by Pascal's Law

\[
P_1 = P_{\text{atm}} + \rho gh
\]  (3)

where \( \rho = 1.0 \times 10^3 \frac{kg}{m^3} \) is the density of water, \( g = 9.8 \frac{m}{s^2} \), and \( h \) is the height of the water level above the hole. Thus \( P_1 - P_2 = \rho gh \).

a) Setting eq.'s (1) and (2) equal yields a differential equation for the rate of change of the height of the water as a function of time

\[
\frac{dh}{dt} = -\frac{\pi (P_1 - P_2) R^4}{8 \eta L A}
\]  (4)

Using Pascal's Law (3) for the pressure difference in equation (4) then gives

\[
\frac{dh}{dt} = -\frac{\pi \rho g R^4}{8 \eta L A}
\]

This is a first order linear differential equation for the height and can be integrated by the techniques of separation of variables:

\[
\frac{\pi \rho g R^4}{8 \eta L A} \int \frac{dh}{h} = \int \frac{dt}{A}
\]

Performing the integration of equation, we have

\[
\ln h(t) - \ln h_0 = -\frac{\pi \rho g R^4}{A 8 \eta L} t
\]  (5)
Now exponentiate both sides of eq. (5) to derive the height as a function of time:

\[ h(t) = h_0 \exp\left(-\frac{\pi \rho g R^4}{A8\eta L}\right) \tag{6} \]

By making a semi-log plot of the height vs. average time, the coefficient of viscosity can be calculated from the slope of the best-fit straight line according to

\[ \eta = -\frac{\pi \rho g R^4}{A8L\text{(slope)}} \tag{7} \]

I. Calculating the Viscosity of Water:

Using eq. (7), calculate your experimental values for the viscosity of water for each length of pipe using the data you obtained in Experiment Flow? What was the temperature of the water for your measurements? The coefficient of viscosity varies as a function of the temperature of the water.