Problem 6: Expt vs results

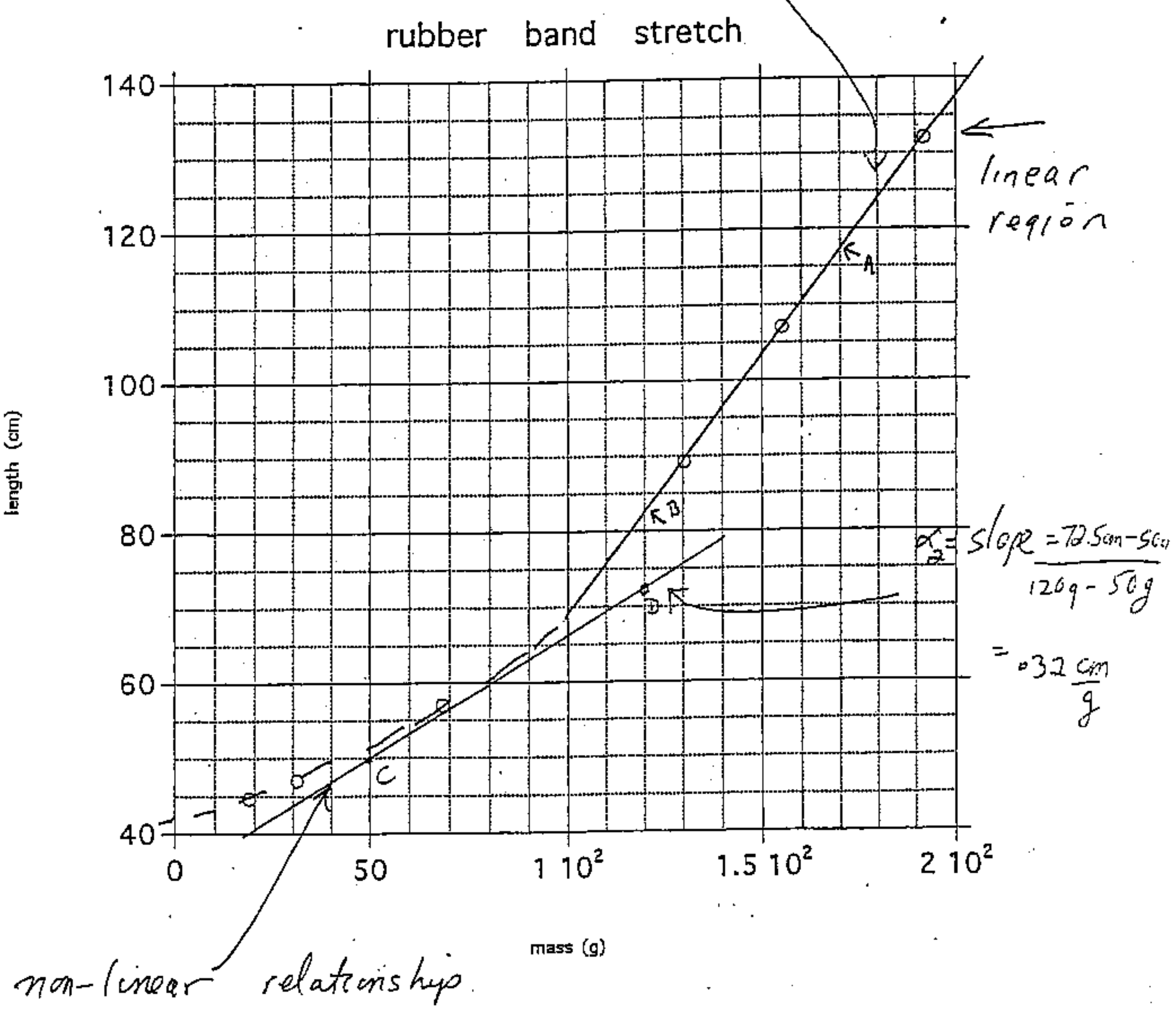
Mass/Spring Oscillator:

1(a) For the chain of rubber bands, plot your data of length of chain of rubber bands vs. weight of hanging pennies. Is your data linear? Determine the force constant (spring constant) in SI units in the neighborhood of 150 g by using a best-fit straight line. Use between 180 g and 120 g as a range for your best-fit line.

mass (g)	length (cm)
18.70	44.6
31.10	47.0
68.30	57.0
130.3	89.2
155.1	107
192.3	132

$x = \alpha m + m_0$

$\alpha_1 = \frac{\Delta x}{\Delta m} = \text{slope} = \frac{117.5\text{ cm} - 82.5\text{ cm}}{170\text{ g} - 120\text{ g}} = 0.7 \frac{\text{cm}}{\text{g}}$



$$\text{Force} = K\,X$$

$$\Delta m g = k \Delta x$$

$$k = \frac{\Delta m g}{\Delta X}$$

the slope of the graph $x = \alpha m + m_0$

$\alpha_1 = \frac{\Delta x}{\Delta m}$ therefore $k = \frac{g}{\text{slope}}$

$$k_1 = \frac{9.8\ m/s^2}{(0.7\ cm/g)\left(\frac{10^3 g}{kg}\right)\left(\frac{1m}{10^2 cm}\right)} = 1.4\ \frac{N}{m}$$

Notice that we only need the slope of the straight line! I also approximate the spring constant for $m = 68.3$ g (20 pennies).

$$\alpha_2 = .32\ \frac{cm}{g} \Rightarrow k_2 = \frac{9.8\ m/s^2}{(0.32\ \frac{cm}{g})\left(\frac{10^3 g}{kg}\right)\left(\frac{1m}{10^2 cm}\right)} = 3.1\ \frac{N}{m}$$

is the approximate spring constant when there were 20 pennies in cup.

2) Qualitatively, how does the force constant change as the load is increased from zero up to the maximum you used? Since $k \sim \frac{1}{\text{slope}}$, and the slope increases with the mass, the spring constant decreases until it reaches a constant value.

3) Describe the motion of the cup when it is both swinging and moving up and down. I tried this for two different weights, 20 pennies and 40 pennies. For 40 pennies, there was very little vertical oscillation. For twenty pennies, the system started to oscillate vertically while it was swinging. Then the vertical oscillations damped out quickly. Then they began again suggesting a 'beating' phenomena. This is a complicated coupled motion with resonance modes but I was not at one of the resonances.

Simple Pendulum with One String:

4)) What are the length and period of your simple pendulum with one string when you put 50 pennies in the cup?

I used a length of 185.1 cm ± 0.5 cm. The initial angle was 10 ^{0}. Here is the data for three runs of ten swings each. The average period was 2.704 s.

$(T_{expt})_{ave} = 2.704$

$(T_{expt})^2_{ave} = 7.31\,s^2$

swings	Time (s)
10	27.13
10	27.10
10	26.89

$T^2_{theory} = 4\pi^2 \frac{l}{g}$

$T^2_{theory} = 7.46$

simple pendulum

swings	Time (s)
10	26.95
10	26.99
10	27.03

$T_{ave} = 2.70\,s$

$(T_{ave})^2 = 7.28\,s^2$

conical pendulum

5) Here is some data for pendulums of shorter length

My longest pendulum was 91.3 cm from the center of mass to the center of the supporting beam. There is an error of ±0.5 cm due to the uncertain location of the actual pivot point on the bar. There is also an error of ±0.3 cm due to the uncertain location of the center of mass. The average period at a 10 ^{0} intial deflection was 1.904 sec. Here is the data for twenty swings; and ten swings for the two shorter lengths

length 91.3 cm

swings	time (s)
20.0	38.0
20.0	38.0
20.0	38.1
20.0	38.2

length 47.0 cm

swings	time (s)
10	13.54
10	13.56
10	13.46

length 19.8 cm

swings	time (s)
10	10.59
10	10.67
10	10.70

) Explain your observations of the effect of different numbers of pennies on the time needed for the amplitude to drop by 50%.

When there were 50 pennies in the cup hanging 91.3 cm from the pivot point; the amplitude decreases by 50% in an average time of 105 sec. When there were 5 pennies in the cup, the amplitude decrease by 50% in an average of 18 sec. The heavier bob has alonger time constant for the decay in amplitude. This suggests that the resistive term $f_{resistive}$ is independent of the mass.

$$ml\,d^2\theta/dt^2 = -mg\theta - f_{resistive}$$

Thus when dividing through by mass the new term $f_{resistive}/m$ decreases as mass increases. the time constant for the decay $\sim$ mass. The data suggests that the relationship is linear. (or nearly linear)

6) Did the period change when you changed the number of pennies in the cup?
When 5 pennies were placed in the cup, the average period was 1.89 s for a length of 91.3 cm. This is nearly in agreement with the result for 50 pennies.

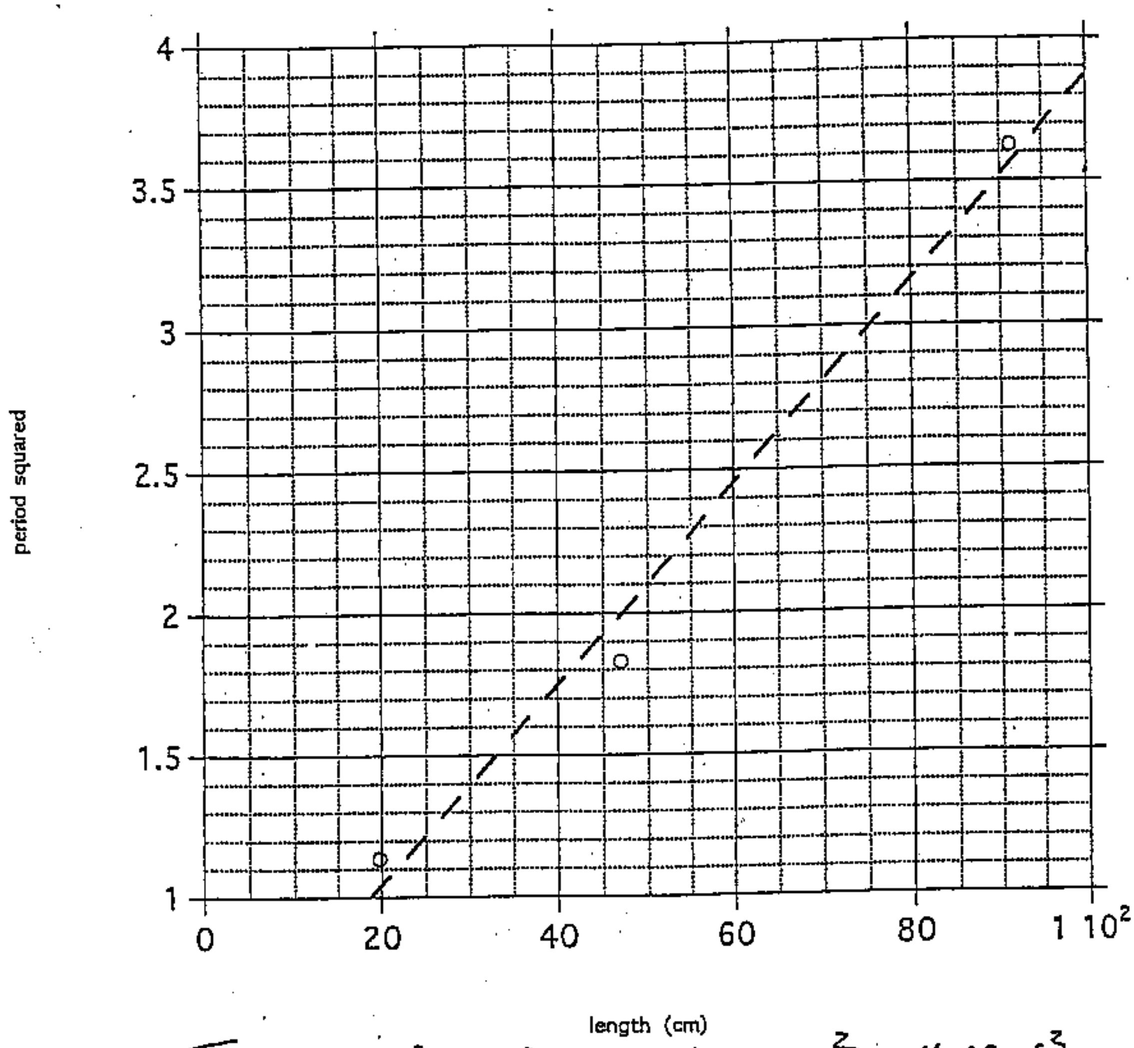
7) Plot the square of the period against the length of this pendulum. Compare with theory.

period squared (s^2)	length (cm)	theory period (s^2)
1.134	19.8	.80
1.828	47.0	1.89
3.624	91.3	3.68

the result for the short length is the least in agreement with theory.

- - - y = 0.331 + 0.0354x R= 0.994

Period Squared vs Length

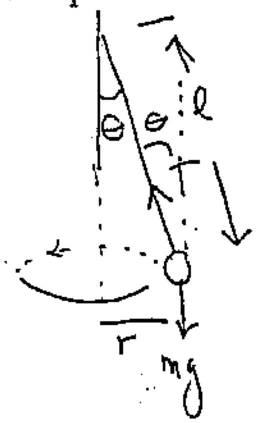


$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2 \frac{l}{g}$, slope $= \frac{4\pi^2}{g} = 4.03 \frac{s^2}{m}$

the data has a slope $= 3.54\, s^2/m$.

8) Are the period, length and orbit radius of the conical pendulum consistent with your theoretical expectation? (Solve theoretically for the period of the conical pendulum as a function of the angle θ the string makes with the vertical, the length L of the string, and g. Remember the cup is undergoing circular motion.)

I used a length of 185.1 cm ± 0.5 cm. The initial angle was 5.56 ⁰.The data for the conical pendulum agrees with simple pendulum (see above data)



$r = l\sin\theta$

$T\cos\theta - mg = 0$ (1)

$-T\sin\theta = -m r \frac{4\pi^2}{T^2}$ (2)

$-T\sin\theta = -m l \sin\theta \frac{4\pi^2}{T^2}$ (2a)

(1) $\Rightarrow T = mg/\cos\theta$

(2a) $\Rightarrow T = m l \cancel{\sin\theta} \frac{4\pi^2}{T^2}$

so $\frac{mg}{\cos\theta} = m l\, 4\pi^2/T^2 \Rightarrow$

$T^2 = 4\pi^2 \frac{l}{g} \cos\theta$

$T^2 = \frac{(4\pi^2)(185.1\ m)}{9.8\ m/s^2} \cos(5.56^\circ) = 7.42\ sec^2$ (theory).

$T^2_{expt} = 7.28\ sec^2$, this is good agreement.