MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01x Fall Term 2001

PROBLEM SET 10 SOLUTIONS

Handed out November 9, due November 16 at 5 pm in 4-339B

Problem 1:

This is a very simple problem masquerading as a complicated one. If you wanted to, you could work through the collision using momentum and energy methods, and you would find that in an equal-mass elastic collision with the second mass inititially stationary like this, the first mass comes to a stop and the second mass flies off with the same initial velocity v.

However, this is not necessary to work the problem. All that is required is to notice that whatever happens in the collision, the same collision will take place in both situations, so the final velocity of the second mass (whatever it happens to be) will be the same. Then it is just a matter of comparing angular velocities, given by:

$$\omega = \frac{v}{r}$$

Since r is twice as large in the second case but the velocity is unchanged, the angular velocity will be half that of the first case.

Problem 2:

This problem is exactly analogous to a one-dimensional linear kinematics problem: a block has an initial velocity $v = 30$ m/s and slides to a halt in 10 seconds; what is the distance it slid through?

a.) We can use the initial angular velocity and the time to find the angular acceleration, here assumed to be constant.

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{-30\ \text{rad/sec}}{10\ \text{sec}} = -3\frac{\text{rad}}{\text{sec}^2}$$

Then we can find the total angle knowing the time and the angular acceleration:

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = 300\ \text{rad} - 150\ \text{rad} = 150\ \text{radians}$$

b.) Since there are 2π radians in a complete circle, the number of revolutions is just:

$$\frac{\theta}{2\pi} = 23.9\ \text{revolutions}$$

Problem 3:

a.) Angular momentum is given by the equation:

$$\vec{L} = m\vec{r} \times \vec{v}$$

Note that as the particle moves, the position vector changes, as does the angle between position and velocity. However, the combination of the two, as given by the cross product, is constant. This can be seen by using one of our alternate methods for computing the cross product: the magnitude of one vector times the perpendicular component of the other vector. Here, we can take the magnitude of velocity, and the perpendicular (i.e. x component) of position, both of which are unchanging in time. So we have:

$$L = mxv = (2\text{ kg})(5\text{ m})(3\text{ m/s}) = 30\frac{\text{kg m}^2}{\text{sec}}$$

The direction is given by the right hand rule: out of the page.

b.) Since this angular momentum is constant, no torque is required to maintain it. This is what would be expected by Newton's first law: no force is required to maintain a constant velocity in a constant direction, as is the case here.

Problem 4:

a.) The torque is simply:

$$\vec{\tau} = \vec{r} \times \vec{F} = (0.12\text{ m})(1\text{ N}) = 0.12\text{ N m}$$

where we have taken the choice of clockwise as the positive sense of rotation.

b.) Angular acceleration is related to torque through the moment of inertia.

$$\tau = I_{disk}\alpha = \frac{1}{2}MR^2\alpha$$

$$\alpha = \frac{2(0.12\text{ N m})}{(1\text{ kg})(0.12\text{ m})^2} = 16.7\frac{\text{rad}}{\text{sec}^2}$$

c.) If we know angular acceleration and time, finding angular velocity is just kinematics:

$$\omega = \alpha\, t = (16.7\frac{\text{rad}}{\text{sec}^2})(0.5\text{sec}) = 8.3\frac{\text{rad}}{\text{sec}}$$

d.) The angular momentum of a rigid object is related to its angular velocity through the moment of inertia:

$$L = I_{disk}\omega = \frac{1}{2}(1\text{ kg})(0.12\text{ m})^2(8.3\frac{\text{rad}}{\text{sec}}) = 0.060\frac{\text{kg m}^2}{\text{sec}}$$

e.) Finding the total angle is once more just a kinematic problem:

$$\theta = \frac{1}{2}\alpha\, t^2 = \frac{1}{2}(16.7\frac{\text{rad}}{\text{sec}^2})(0.5\text{sec})^2 = 2.08\text{ radians}$$