Solutions for 8.01x, homework 11 (Fall 2001)

1 Problem 1

For the angular momentum of the ball, we get $L = r \cdot J_0 \cdot \sin\Theta = r \cdot J_0 \cdot (h-r)/r$, where Θ is the angle between the horizontal and the line connecting the point of impact and the center of the ball. Using $L = I \cdot \omega_0 = \frac{2}{5} \cdot m \cdot r^2 \cdot \omega_0$, we get

$$\omega_0 = \frac{5}{2} J_0/m \cdot (h-r)/r^2 \tag{1}$$

For the linear momentum gives

$$p = m \cdot v_0 = J_0 \tag{2}$$

To express w_0 as a function of v_0, h, r, we use $v_0 = \frac{J_0}{m}$, and get

$$w_0 = \frac{5}{2} v_0 (h-r)/r^2 \tag{3}$$

1.1 b

For the ball not to slip, we need $w_0 = v_0/r$. Inserting this in equation 3 gives

$$v_0/r = \frac{5}{2} v_0 (h-r)/r^2 \tag{4}$$

which leads to $\underline{7/5 \cdot r = h}$.

2 Problem 2

2.1 a

The moments of inertia for the hoop, disk and spere are:

$$\begin{aligned} I_{hoop} &= M \cdot R^2 \\ I_{disk} &= 0.5 \cdot M \cdot R^2 \\ I_{sphere} &= 0.4 \cdot M \cdot R^2 \end{aligned}$$

For each object, the kinetic energy at the bottom is equal to the potential energy at the top of the incline:

$$E_{kin} = 0.5 \cdot m \cdot v^2 + 0.5 \cdot I \cdot \omega^2 = m \cdot g\ h. \tag{5}$$

Using the condition that the objects are rolling without slipping, i.e. $w = v/R.$, this means

$$0.5 \cdot m \cdot v^2 + 0.5 \cdot I \cdot v^2/R^2 = m \cdot g\ h. \tag{6}$$

This gives

$$v = \sqrt{(2 \cdot g \cdot h)/(1+k)}, \tag{7}$$

where $k = 1, 0.5, 0.4$ for the hoop, disk and sphere respectively. For $g = 9.8 m/s^2$ and $h = 2$ m, this gives $v = 4.43 m/s, 5.11 m/s, 5.29 m/s$ for hoop, disk and sphere respectively.

2.2 b

The torque by the force of friction gives the angular acceleration:

$$\tau = F_f \cdot R = I \cdot \alpha \tag{8}$$

Rolling motion without slipping requires $\omega \cdot R = v$ and therefore $\alpha \cdot R = a$. Therefore

$$F_f \cdot R = I \cdot a/R \tag{9}$$

The linear acceleration is given by

$$a = F/m = (m \cdot g \cdot \sin\Theta - F_f)/m \tag{10}$$

Therefore,

$$F_f \cdot R = I \cdot (m \cdot g \cdot \sin\Theta - F_f)/(m \cdot R) \tag{11}$$

or

$$F_f = k/(1+k) \cdot m \cdot g \cdot \sin\Theta \tag{12}$$

giving 7.35 N, 4.9 N, 4.2 N for hoop, disk and sphere respectively.

2.3 c

The total time is given by the average velocity $v_{ave} = 0.5 \cdot v_{final}$ and the total distance $s = h/\sin\Theta = 4$ m. This gives 1.81 s, 1.56 s and 1.51 s for hoop, disk and sphere.

3 Problem 3

3.1 a

The original distance of the spaceship from the planet is determined by the fact that the centripetal acceleration is given by the gravitational attraction between spaceship and Venus:

$$m \cdot v_0^2/R_0 = G\frac{m \cdot M}{R^2} \tag{13}$$

which leads to

$$R_0 = \frac{G \cdot M}{v_0^2} \tag{14}$$

3.2 b

After firing the engine the spaceship will enter an ellipsoidal orbit that touches the original circular orbit at the point where the engines were fired.

3.3 c

After firing the engine, the total energy of the spaceship is

$$E = 1/2 \cdot m \cdot (1.25 v_0)^2 - G \cdot m \cdot M/R_0. \tag{15}$$

At the point of maximal distance to Venus, the energy is

$$E = 1/2 \cdot m \cdot v_{R_{max}}^2 - G \cdot m \cdot M/R_{max}. \tag{16}$$

An additional constraint can be obtained from angular momentum conservation

$$m \cdot 1.25 \cdot v_0 \cdot R_0 = m \cdot v_{R_{max}} \cdot R_{max}. \tag{17}$$

Rewriting this

$$1.25 \cdot v_0 \cdot R_0/R_{max} = v_{R_{max}}. \tag{18}$$

Using energy conservation to set 15 and 16 equal, and inserting 18 into this, we have an equation for R_{max} with just one unknown.

4 Problem 4

4.1 a

We start from

$$\tau = I\alpha \tag{19}$$

This gives (using $\sin(\Theta) \approx \Theta$ for small Θ):

$$-m \cdot g \cdot L \cdot \Theta = I \frac{d^2\Theta}{dt^2} \tag{20}$$

I is the total momentum of inertia of motion of the CM ($m \cdot L^2$) *and* the rotation of the sphere around the CM ($2/5 \cdot m \cdot r^2$). Using an ansatz $\Theta(t) = A\cos(\omega t + \phi)$ and inserting this in equation 20, gives

$$\omega = \sqrt{m \cdot g \cdot L/I} = \sqrt{m \cdot g \cdot L/(m \cdot L^2 + 2/5 \cdot m \cdot r^2)}. \tag{21}$$

We know $T = 2\pi/\omega$, so

$$T = 2\pi\sqrt{\frac{L + 2/5 \cdot R^2/L}{g}} = 2\pi\sqrt{L/g}\sqrt{1 + \frac{2r^2}{5L^2}}. \tag{22}$$

4.2 b

The second term $\sqrt{1 + \frac{2r^2}{5L^2}}$ is 1.00008 for $r = 0.02$m and $L = 1$m. So the error is very small (< 0.01 %).

4.3 c

For the correction factor to be ≈ 1.01, r has to be 0.22 m (for $L = 1$ m).