MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01x Fall Term 2001

# PROBLEM SET 12 SOLUTIONS

Handed out November 30, due December 7 at 5 pm in 4-339B

**Problem 1:**

**a.)** The maximum force will just be the maximum pressure times the effective cross-sectional area of the femur, obtained from its diameter:

$$F = P\pi r^2 = (1.50\times 10^8 \frac{\mathrm{N}}{\mathrm{m}^2})\,\pi\left(\frac{0.025\text{ m}}{2}\right)^2$$

$$F = 74,000\text{ N}$$

**b.)** Using the Young's modulus, we can relate the stress on the bone (which is already given in our number for maximum pressure) to the strain it undergoes:

$$Y\frac{\Delta x}{x} = \frac{F}{A}$$

$$\Delta x = x\frac{1}{Y}\frac{F}{A} = (25.0\text{ cm})\left(\frac{1.50\times 10^8\text{ N/m}^2}{1.50\times 10^{10}\text{ N/m}^2}\right)$$

$$\Delta x = 0.25\text{ cm}$$

**Problem 2:**

We can use Bernoulli's principle to relate the pressure and velocity of the fluid in the fat and thin sections of the pipe, under our usual assumptions of incompressibility, steady flow, and lack of viscosity:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

We can also use the continuity equation to relate the velocities to the cross-sectional areas:

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

**a.)** At the exit of the pipe, the water is flowing with speed $v_2$ at atmospheric pressure $P_0$. Therefore the pressure in the fat section of tube is given by:

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 = P_0 + \frac{1}{2}\rho\left(\frac{A_1}{A_2}v_1\right)^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 = P_0 + \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

Now, using Pascal's law, we can relate the pressure $P_1$ in the bottom of the offshoot tube to its height *h*. Note that we cannot simply use Bernoulli's equation again for this, since the offshoot tube is not part of the same steady flow: it is only the pressure at the bottom of the tube that is the same, not the combination of pressure and velocity.

$$P_1 = P_0 + \rho g h$$

$$h = \frac{v_1^2}{2g}\left(\frac{A_1^2}{A_2^2} - 1\right)$$

**b.)** For the second offshoot tube, we can immediately see that the water would not rise at all, since the water is already flowing exactly at atmospheric pressure in the thin section of the pipe, the same as the pressure at the top of the column of fluid would need to be. This conclusion is only valid for fluids without viscosity, though. When viscosity is added, there will be a finite pressure difference along the length of the pipe, which would cause water to rise into the offshoot tube.

**Problem 3:**

Note that the original problem had a typo in it: *M* should be 0.6 kg instead of 6.0 kg. If you did not catch the correction in working the problem, you would arrive at a nonsensical answer (a negative density for the air).

**a.)** The system is static, so we can use force balance between gravity and the buoyancy force to obtain the density of the air inside the sphere:

$$\sum F = 0$$

$$-(M + \rho_{gas}V)g + \frac{2}{3}\rho_{water}Vg = 0$$

$$\rho_{gas} = \frac{2}{3}\rho_{water} - \frac{M}{V}$$

$$\rho_{gas} = \frac{2}{3}(1000\frac{\text{kg}}{\text{m}^3}) - \frac{3(0.6\,\text{kg})}{4\pi(0.10\,\text{m})^3} = 523\frac{\text{kg}}{\text{m}^3}$$

**b.)** It will float exactly the same as before. Since the sphere is sealed, no air can escape, so its density and mass are still the same. The only difference is that the internal pressure in the sphere is higher due to the temperature, but this does not affect the buoyancy forces at all, assuming the sphere is rigid enough not to expand significantly with a higher internal pressure.