Problem Set 1 Solutions. Physics 8.01x, Fall 2001

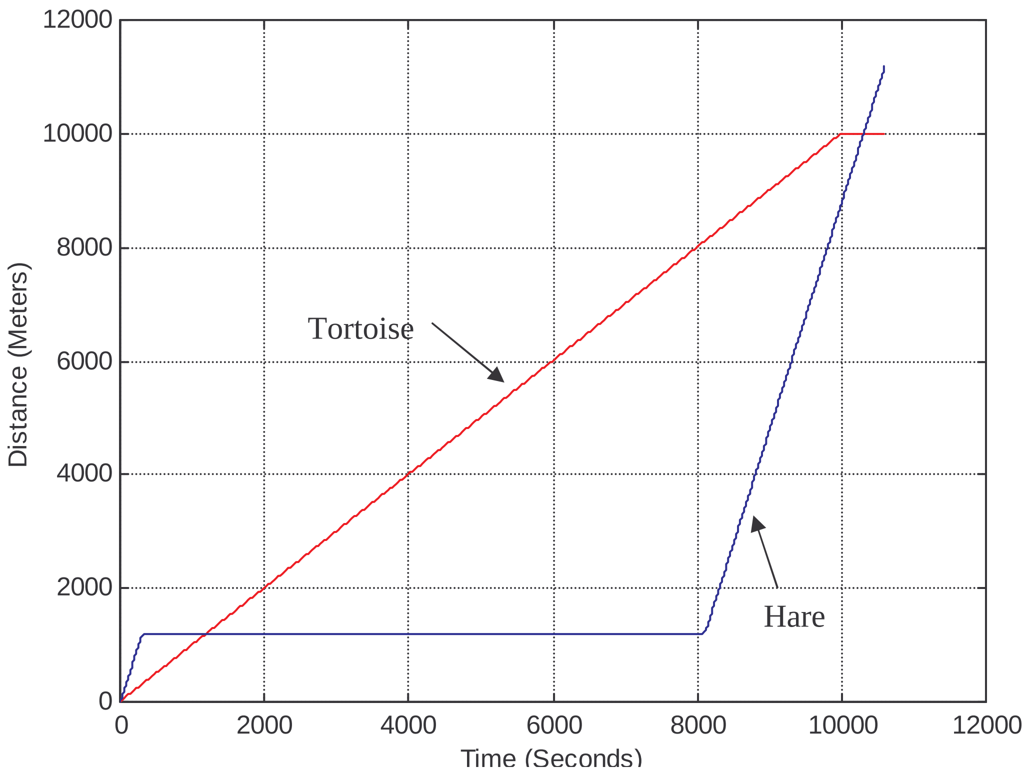
Problem 1

a) FALSE. Average and instantaneous velocities may indeed be equal. Consider 2 cases: (1) Imagine that you have a perfect cruise control in your car and you drive down a straight highway for an hour. The cruise control keeps your speed constant. So, your velocity at any instant during the hour (your instantaneous velocity) will be equal to your average velocity. (2) Turn of the cruise control, drive for an hour, and allow the car to speed up and slow down. At the end of the hour, you've traveled a certain distance. Divide the distance (actually the vector between the start and end points) by the time (one hour) to get the average velocity. It may certainly happen that at some instant during the trip, you happened to be traveling with an instantaneous velocity that is equal to this average velocity. Nothing forbids this.

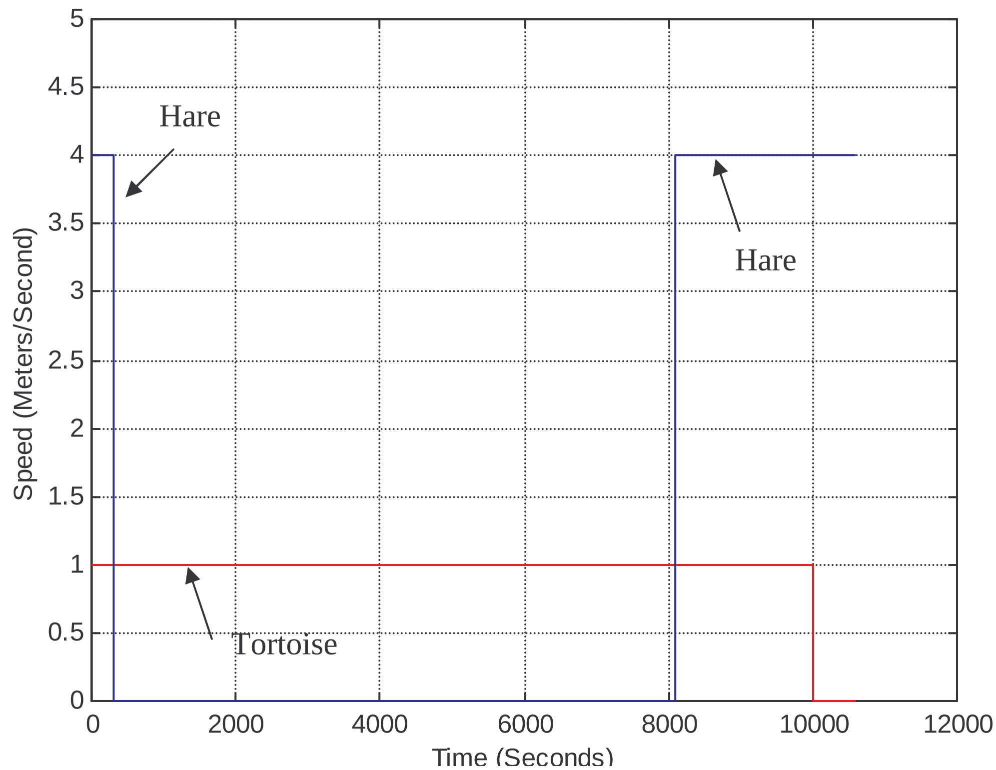
b) FALSE. If the average velocity is non-zero during a specific time interval, the instantaneous velocity may certainly be zero during this time interval. For instance, during an hour long trip, you could stop your car for a few minutes and then start up again. At the end of the hour, you would have a nonzero average velocity, but you had an instantaneous velocity of zero while your car stopped.

c) FALSE. Imagine that you drove your car around in a circle and you ended up back where you started. You traveled zero distance, so your average velocity was zero, but your instantaneous velocity was always nonzero.

Problem 2a)



Problem 2b)



2c) See graph (a) above. The tortoise passes the hare when t=1200 seconds.

2d) The tortoise crosses the finish line at t=10000 seconds. At this time, the hare is at 1200 meters+1900seconds*(4 meters/second)=8800 meters.

2e) At t=10000 seconds, the hare needs to be 1200 meters ahead of where he ends up now. This means that he should start running 300 seconds earlier. His original nap was 130 minutes long. So, he needs to nap less than 125 minutes to win.

Problem 3)

a) 2 slices=0.5 square feet. A typical undergraduate then eats 0.5 square feet of pizza per week (Note from the surgeon general: more isn't healthy). There are about 50 weeks in a year. So we get 25 square feet per undergraduate. There are around 5000 undergraduates at MIT, so we get 125,000 square feet. There are nine square feet in a square yard, so this gives us around 14,000 square yards.

b) An apple is about 10 centimeters (0.1 meters) in diameter. The volume of the apple is given by $(4/3)\pi R^3$, where R is the radius of the apple (0.05 meters). The volume of a gold atom is given by $(4/3)\pi r^3$. Here r is the radius of the atom $(1.5\text{x}10^{-9})$ meters. The number of gold atoms in the apple is about:

$$\left(\frac{0.05\ meters}{1.5\times10^{-9}\ meters}\right)^3 = 4\times10^{22}\text{ atoms.}$$

Considering a 100 atom thick foil (300 nanometers thick) , these atoms could be spread over:

$$Area = \frac{Volume\ per\ atom}{thickness\ of\ 100\ atoms}\times\#of\ \ atoms$$

Performing this calculation, we get about 2000 square meters. Considering that one bust of Newton takes about one square meter of gold, we could produce 2000 busts of Newton. Notice that you don’t need information about the density of gold to solve this problem.

c) The distance to the nearest star from the sun is about 4 light-years. Taking this number to be typical in our galaxy, find the approximate number of stars in the galaxy.

The Milky Way as a short cylinder with volume $\pi R^2 h$, where R is the radius (10^{21} meters) and h is the thickness (10^{19} meters). Converting these distances to light years (1 light year = 300,000,000 meters/second*3600seconds/hour*24hours/day*365days/year$\approx10^{16}$meters) we get $R=10^5$ light-years and $h=10^4$ light-years. The galaxy’s volume is then about $3\text{x}10^{14}$ cubic light-years. Consider that each star has to itself a volume of around $4/3\left(\pi r^3\right)$ where r is the typical distance to the nearest neighboring star. Taking r equal to 4 light years, we get a typical volume per star of about 300 cubic light-years. Dividing the galaxy volume by the volume per star, we get about 10^{12} stars (one trillion) in the galaxy.