MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01x Fall Term 2001

PROBLEM SET 2 SOLUTIONS

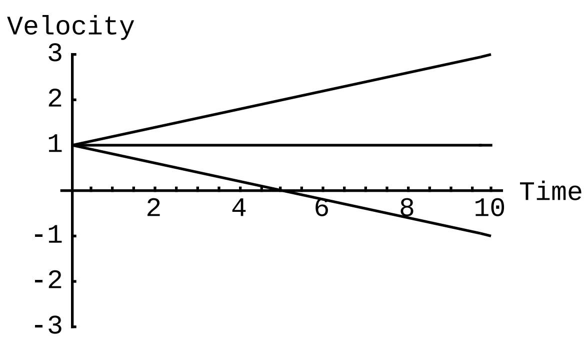
Handed out September 14, due September 21 at 5 pm in 4-339B

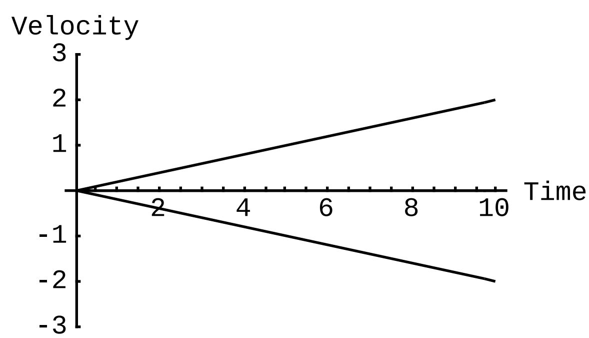
Problem 1:

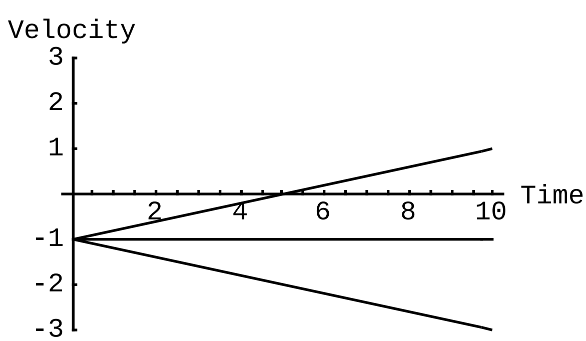
The nine cases can be summarized as follows. We take eastward to be the positive direction of our coordinate system:

$v(t=0)$	a	Description
+	+	traveling east and speeding up
+	0	traveling east at a constant speed
+	-	traveling east, slowing down, turning around, and speeding up
0	+	starting from rest and heading east
0	0	standing still
0	-	starting from rest and heading west
-	+	traveling west, slowing down, turning around, and speeding up
-	0	traveling west at a constant speed
-	-	traveling west and speeding up

We can plot sample velocity graphs for these cases. Here they are given in groups of three, ordered top to bottom as in the above table, with one graph for each choice of initial velocity. The vertical and horizontal scales are of course arbitrary:





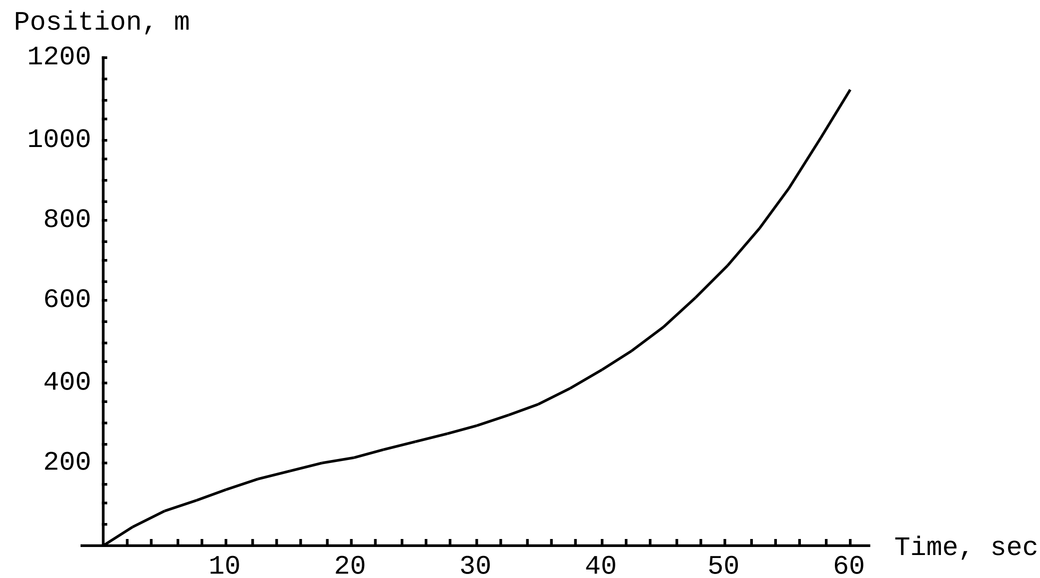


The acceleration plots are simply constant horizontal lines of positive, zero, or negative value, so they are not printed here.

Problem 2:

Note that the graph as originally given in the printed homework does not exactly match the equation. See the online copy of problem set 2 for an updated graph. The two graphs are similar in shape except that the original homework had the velocity drop to zero, but in the revision it levels off at a small but positive value.

a.) Qualitatively, we expect the position graph to start at some initial position (taken to be zero) and begin rising in the positive direction, gradually leveling off as the velocity drops, and then curving upwards again as the velocity once more increases. Using the derived equation from part (b), we can show this exactly:



A qualitative sketch based on the original homework graph (as opposed to the equation) would show the position curve leveling off to be horizontal momentarily at a time of 25 seconds, instead of retaining an upwards slope throughout the interval.

b.) To compute position from velocity, we can integrate the velocity equation:

$$v(t) = \frac{dx}{dt}$$

$$\int v(t)\, dt = \int dx = x(t)$$

We will take the x position to be zero when t is zero, to fix the constant of integration. Integrating our equation for velocity, using the familiar rules for integration of polynomials, gives:

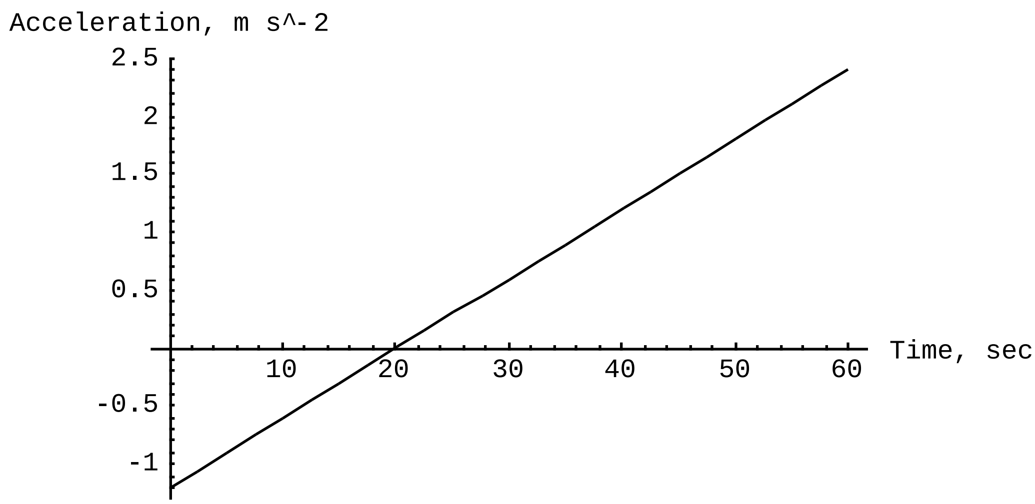
$$x(t) = \int_0^t 0.03\,(s^2 - 40s + 625)\,ds$$

$$x(t) = 0.03\left(\frac{1}{3}t^3 - 40 \cdot \frac{1}{2}t^2 + 625t\right)$$

$$x(t) = 0.01t^3 - 0.6t^2 + 18.75t$$

Note that for clarity we have used s as the independent variable inside the integral instead of t, since t is already used as the upper integration limit.

c.) Qualitatively, we expect the acceleration, as determined from the slope of the velocity graph, to start negative, then pass through zero and become positive. Whether this is a curved or a straight line might not be immediately obvious from a graphical analysis, but from the equations we can see that if velocity is a quadratic function, then acceleration must be at most linear. So a straight-line graph is correct:



A sketch made from the original homework graph would have its zero at t=25 sec instead of 20 sec, since that is the location of the velocity minimum.

d.) We can obtain an exact equation for the acceleration by differentiating the velocity equation, using our well-known rules for the derivative of a polynomial:

$$a(t) = \frac{dv}{dt}$$

$$a(t) = \frac{d}{dt}\left(0.03\left(t^2 - 40t + 625\right)\right)$$

$$a(t) = 0.03\,(2t - 40) = 0.06t - 1.2$$

Problem 3:

Under the influence of gravity, an object will fall with an acceleration given by g=9.8 m/s^2. For the purposes of this problem, we will assume a coordinate system with an origin at ground level and positive being upwards. Our origin in time (t=0) is taken to be when the crate is released from the helicopter.

a.) If a crate is dropped from rest, its position and velocity are given by:

$$y(t) = h - \frac{1}{2}gt^2$$
$$v(t) = -gt$$

where h is the initial height of the helicopter. The eggs will crack at a limiting velocity of 8 m/s, so we can solve for the time this would occur from the second equation. Note that since this is a downward velocity, it should be negative according to the conventions of our coordinate system.

$$-8\text{ m/s} = -(9.8\text{ m/s}^2)t$$
$$t = 0.816\text{ sec}$$

Plugging this in to our position equation lets us solve for the initial height, since we know that y=0 at the time of impact:

$$0 = h - \frac{1}{2}(9.8\text{ m/s}^2)(0.816\text{ sec})^2$$
$$h = 3.27\text{ m}$$

b.) If we now give the crate an initial upward velocity of 3 m/s (from the motion of the helicopter as the crate is being released) and drop it from a height of 9 m, the equations of motion become:

$$y(t) = 9\text{ m} + (3\text{ m/s})t - \frac{1}{2}gt^2$$
$$v(t) = 3\text{ m/s} - gt$$

We can use the first equation to solve for the time that the crate hits the ground, since y=0 at that point. Using the quadratic formula, we get:

$$0 = 9\text{ m} + (3\text{ m/s})t - (4.9\text{ m/s}^2)t^2$$
$$t = 1.70\text{ sec} \quad \text{or} \quad t = -1.08\text{ sec}$$

The two solutions correspond to the time at which the box hits the ground when dropped from that height and that velocity at t=0, and the time in the past at which the box would have had to be thrown off the ground to reach that height and that velocity by t=0. For our situation, the positive solution is the relevant one.

c.) We can use the velocity equation above to solve for the impact speed, since we know the time of impact:

$$v_f = 3\text{ m/s} - (9.8\text{ m/s}^2)(1.70\text{ sec}) = -13.6\text{ m/s}$$

The eggs will of course be cracked, but the impact velocity is less than the tomato-squashing limit of 16 m/s, so those will still be okay. The final velocity is negative as expected, since that is the downwards direction in our coordinate system.

Problem 4:

The easiest way to approach this problem is by breaking the vectors into components. This is not strictly necessary, but the trigonometry to avoid it gets rather complicated.

a.) We set up a standard *x*-*y* coordinate system with positive directions to the right and upwards, respectively. The three forces can be broken down into their components:

$$\vec{F}_1 = (7.0\text{ N})(-\hat{i}\cos 60^{o} + \hat{j}\sin 60^{o}) = -3.5\hat{i} + 6.06\hat{j}\text{ N}$$
$$\vec{F}_2 = (3.0\text{ N})(\hat{i}\cos 45^{o} + \hat{j}\sin 45^{o}) = 2.12\,\hat{i} + 2.12\,\hat{j}\text{ N}$$
$$\vec{F}_3 = (6.0\text{ N})(-\hat{j}) = -6.0\,\hat{j}\text{ N}$$

Note that we have explicitly inserted a minus sign into the expression for the *x* component of the first force, rather than convert the angle to 120^{o} from the *x* axis, although both techniques would work. The third force uses a similar shortcut since it it directed exactly along the negative *y* axis. We can just sum the forces by their components now:

$$\vec{F}_{total} = (-3.5 + 2.12)\hat{i} + (6.06 + 2.12 - 6.0)\hat{j}\text{ N}$$
$$\vec{F}_{total} = -1.38\hat{i} + 2.18\hat{j}\text{ N}$$

Now we can use a little trigonometry to reconstruct the magnitude and direction of this force:

$$|F_{total}| = \sqrt{1.38^2 + 2.18^2}\text{ N} = 2.58\text{ N}$$
$$\theta = \arctan\left(\frac{2.18\text{ N}}{-1.38\text{ N}}\right) = 122^{o}\text{ from the } x \text{ axis}$$

Note that the arc tangent has an ambiguity of 180^{o} in it: the angle could be either 122^{o} or -58^{o} and it would have the same quotient for a tangent. To determine which angle it is, we need to examine the signs of the two components. Since the *x* component is negative and the *y* component is positive, the vector must be somewhere in the upper left quadrant of the coordinate system, so we choose the 122^{o} answer.

b.) Using Newton's second law, and knowing that the mass is 3 kg, we can easily compute the magnitude of the acceleration:

$$\vec{F}_{total} = m\,\vec{a}$$
$$|a| = \frac{|F_{total}|}{m} = \frac{2.58\text{ N}}{3\text{ kg}} = 0.860\text{ m/s}^2$$

The direction of the acceleration is the same as the direction of the total force: 122^{o} from the *x* axis, since Newton's second law relates the force vector and the acceleration vector, not just their magnitudes. The fact that the mass was originally at rest is irrelevant to the acceleration.

Problem 5: (bonus problem)

a.) The reading on the scale is just the combined weight of your mass and the mass you are holding. To be precise, the reading on the scale is the force that your feet are exerting on the scale, which is equal to the gravitational weight when you're standing still:

$$F = mg = (20\text{ kg} + 70\text{ kg})\,(9.8\text{ m/s}^2) = 882\text{ N}$$

The reason that the reading is equal to your weight is a combined function of Newton's second and third laws. You are standing still (acceleration of zero), which by the second law means that the net force on you is zero. The two forces acting on you are the Earth's gravitational attraction (882 N downward) and the force that the scale exerts on the bottom of your feet (which therefore must be 882 N upward). The two forces cancel out to give you zero acceleration. By the third law, the force exerted by the scale on your feet (882 N upward) is equal and opposite to the force exerted by your feet on the scale (therefore 882 N downward), which is what the scale is reading.

Note that gravitational forces and contact forces are *not* third-law pairs of each other, even though in this case they are equal and opposite. They are second-law pairs instead, coming from the zero net force condition of a static system. The third-law pair force of the gravitational attraction of the Earth on you is the equal and opposite gravitational attraction of you on the Earth, which isn't relevant in most problems.

b.) When you lift the mass upward, the reading on the scale increases. To see why this is the case, let's track the forces through.

While holding a 20 kg mass still, your hand has to exert an upward force of 196 N on it, which means by the third law that it is exerting a 196 N downward force on your hand. Simultaneously, gravity is pulling you down with a force of 686 N, and pulling the mass down with a force of 196 N. The scale is pushing up on your feet with a force of 882 N. By adding the forces on the mass (+196 N - 196 N) and all the forces on you (-196 N - 686 N + 882 N) together, we see that neither you nor the mass has any net force on it, so has no acceleration.

To lift the mass you're holding, however, requires you to give it a net upward force. So your hand now exerts, say, a 200 N force on it, while gravity still pulls down at 196 N, so it accelerates upward with a net force of 4 N. By the third law, the mass is now pushing down on your hand with 200 N. Gravity is still pulling you down with 686 N. To ensure that you don't move (net force on you is zero), this means that the scale has to now push up on your feet with 4 N more than before: 886 N in total. By the third law, this means your feet push down on the scale with 886 N as well, which is the reading on the scale, increased from the static case.

c.) The case of throwing the mass into the air is very similar to part (b). While you are accelerating the mass upwards during the process of throwing it, the reading on the scale must increase for the same reason as before. Once it leaves your hands, though, the reading will drop down to reflect only your weight of 686 N.