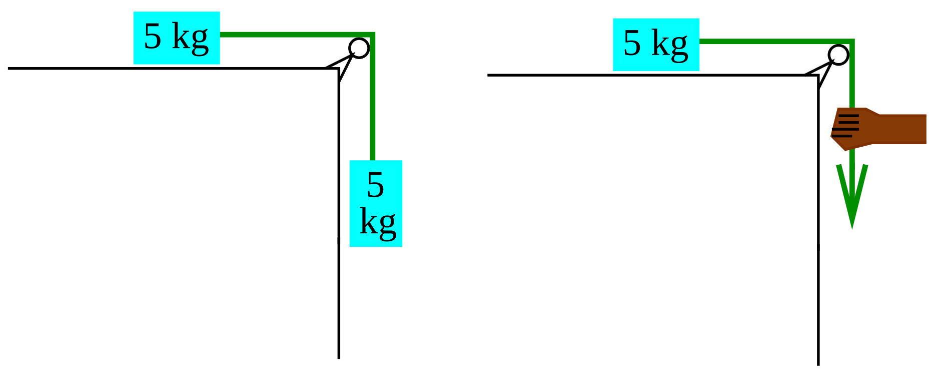
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X — Fall Term 2001

Solutions to PROBLEM SET 3

Problem 1 (10 points):

Two 5 kg bodies are connected by a massless string, as shown in the figure at left. The table is frictionless, and the string passes over a frictionless pulley.



a. Find the acceleration of the masses.

Assume that the magnitude of the tension in the string is T, it is pointing horizontally towards right for the body on the table, and it is pointing vertically up for the body which is hanging.

We can write down the following two equations:

$$Mg - T = Ma \qquad (2pts) \tag{1}$$

$$T = Ma \qquad (2pts) \tag{2}$$

Eqs. (1) + (2) gives:

$$Mg = (M + M)\ a \rightarrow a = \ g/2 = 4.9\ m/s^2 \qquad (1pt) \tag{3}$$

b. Find the tension in the string.

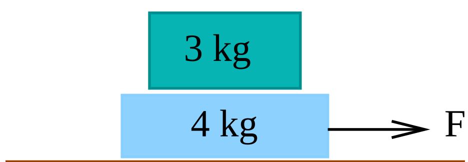
From the acceleration obtained about and plug the number into Eq. (2), one gets the magnitude of the tension: $T = M \times a = 24.5$ N (2pts).

c. Suppose that instead of a 5 kg hanging mass, a person pulls on the string with a force equivalent to the weight of a 5 kg object. Do your answers to parts a. and b. change? Explain.

The tension in the string will be $5kg \times 9.8m/s^2 = 49$ N (2pts), and the acceleration of the block on the table is just $g = 9.8$ m/s^2 (1pt).

Problem 2 (10 points):

A 3 kg block rests on top of a 4 kg block, which rests on a frictionless table. The coefficient of friction between the blocks is such that the blocks start to slip when the horizontal force F applied to the lower block is 24 N. Suppose the horizontal force is now applied only to the upper block. What is its maximum value for the blocks to slide without slipping relative to each other?



Assume the magnitude of the friction between the two blocks is f at the moment just before the blocks start to slip. According to Newton's third law, the direction of the frictional force on the bottom block ($M_1 = 4$ kg) is pointing horizontally towards left and the frictional force on the top block ($M_2 = 3$ kg) is pointing horizontally towards right. We can write down the following two equations:

$$F - f = M_1 a \qquad \text{(1point)} \tag{4}$$

$$f = M_2 a \qquad \text{(1point)} \tag{5}$$

Adding Eqns. (4) and (5), one gets:

$$F = (M_1 + M_2)a \rightarrow a = 3.43\ m/s^2 (1point) \tag{6}$$

Then from Eqn. (5), we can obtain that $f = M_2 \times a = 10.3$ N (1 point).

To find the answer for the maximum force that can be applied to the top block without causing slipping between the two blocks, we can write the following equations:

$$F - f = M_2 a' \qquad \text{(2points)}, \tag{7}$$

$$f = M_1 a', \qquad \text{(2points)} \tag{8}$$

where $f = 10.3$ N. So from Eqn. (8), one gets $a' = 2.575\ m/s^2$ (1 point). Plugging the value of a' into Eqn. (7), we get $F = 18.0$ N (1 point).

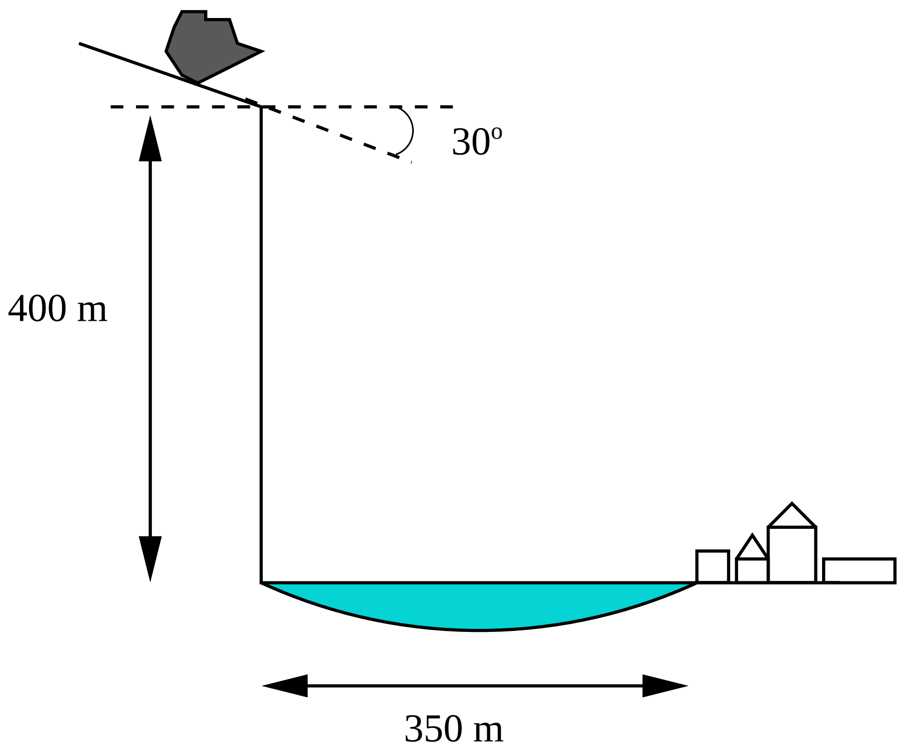
Problem 3 (10 points):

Imagine that you are the mayor of a small village in a scenic mountain valley. A large boulder is positioned on a cliff 400 m above your town, such that if it should roll off, it would leave the cliff with a speed of 50 m/s at an angle of 30 degrees from the horizontal. The town is 350 m away from the base of the cliff, with a lake in between. A geologist tells you that the region has recently become more seismically active, and an earthquake has a significant chance of dislodging the boulder over the next 10 years.

a. Will the boulder hit the village when it lands? Will it hit the lake? (In other words, should you evacuate your citizens?) You can assume that the size of the boulder is negligible compared to either the height of the cliff or the distance between the village and the cliff.

At $t = 0$, the horizontal component of the velocity $V_x^0 = 50cos30^* = 43.3$ m/s (pointing horizontally towards right), and the vertical component of the velocity $V_y^0 = V_o = 50sin30^* = 25$ m/s, which is pointing vertically down (1 point). We can then write down the following equation for the vertical motion:

$$h = V_o t + \frac{1}{2} g t^2, \qquad \text{(2points)} \tag{9}$$



where $h = 400$ meters, g is the gravitational acceleration constant and t is the time when the boulder hits the ground. Solving Eqn. 9, we get $t = 6.84$ seconds (1 point). So when the boulder hits the ground, the horizontal distance it traveled is $x = V_x^0 t = 43.3 * 6.84 = 296.0$ meters (1 point). So it will hit the lake, but not the village when it lands (1 point).

b. How fast will it be going when it hits the ground?

When it hits the ground, the vertical component of the velocity $V_y = V_o + gt = 25.0 + 9.8 \times 6.84 = 92$ m/s (1 point). The magnitude of the net velocity is $V = \sqrt{V_y^2 + V_x^2} = \sqrt{92^2 + 43.3^2} = 101.7$ m/s when it hits the ground (1 point).

c. What will be the horizontal component of the boulder's velocity when it hits the ground?

The horizontal component of the boulder's velocity is a constant, which is 43.3 m/s (1 point).

d. How long will the boulder be in the air?

The boulder will be in the air for 6.84 seconds (1 point).

Problem 4 (10 points):

Re-evaluate your answers to parts a. through d. of Problem 3 under the assumption that the village is actually a colony on Mars, where the acceleration of gravity is 3.7 m/s^2. You can assume the velocity of the boulder as it leaves the cliff is still 50 m/s.

We can solve the problem in exactly the same way as what has been done in Problem 3. For a gravitational acceleration of 3.7 m/s^2, we will find that the amount of the time that the boulder will be in the air is $t = 9.46$ seconds (2 pts). So the horizontal distance that the boulder will travel before it hits the ground is $x = 43.3 \times 9.46 = 409.6$ (2 pts) meters. So it will hit the village in this case (1 point). The horizontal component of the boulder velocity is 43.3 m/s (2 pts) when it hits the ground and it will be in the air for 9.46 seconds before it hits the ground. The magnitude of the net velocity of the boulder when it hits the ground will be $\sqrt{43.3^2 + (25 + 3.7 \times 9.46)} = 74$ m/s (3 points).