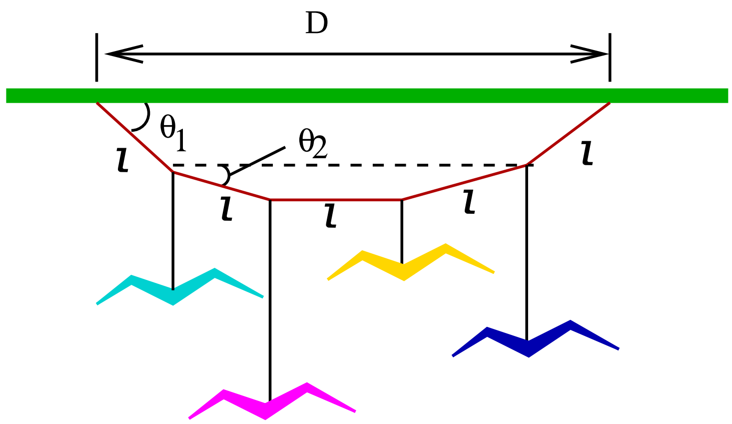
Solutions for 8.01x, homework 4 (Fall 2001)

1 Problem 1

Questions a. and b. need to be solved simultanously. Taking the symmetry of the problem into account, we have a total of 5 unknowns: The tensions T_1, T_2, T_3 in the first, second and third of the 5 strings (starting from the left, see drawing) and the two angles Θ_1 and Θ_2. The tensions in the forth and fifth string will be equal to those in the second and first string, respectively, due to the symmetry of the problem.



Questions a. and b. require us to express each of the unknowns in terms of Θ_1. To derive these expressions we need 4 equations. These can be obtained by setting up the force balance at the two leftmost points where the weights are attached to the strings, looking at the horizontal and vertical forces at each of these two points. In equilibrium the total force has to vanish at each of the two points, i.e. the string tensions have to balance the weight of the mass hanging from this point. For the first point we get:

$$\sum F_{1_x} = 0 = T_1 \cos\Theta_1 - T_2 \cos\Theta_2 \tag{1}$$
$$\sum F_{1_y} = 0 = mg - T_1 \sin\Theta_1 + T_2 \sin\Theta_2 \tag{2}$$

Balancing the forces at the second point gives:

$$\sum F_{2_x} = 0 = T_2 \cos\Theta_2 - T_3 \tag{3}$$
$$\sum F_{2_y} = 0 = mg - T_2 \sin\Theta_2 \tag{4}$$

Adding equations (2) and (4) and rearranging (4) gives:

$$T_1 = \frac{2mg}{\sin\Theta_1} \tag{5}$$
$$T_2 = \frac{mg}{\sin\Theta_2} \text{ and} \tag{6}$$
$$T_3 = \frac{mg}{\tan\Theta_2} \tag{7}$$

Inserting (5) and (6) into (1), we see how the two angles are related (Answer to **question a.**):

$$
\begin{aligned}
0 &= \frac{2mg}{\sin\Theta_1}\cos\Theta_1 - \frac{mg}{\sin\Theta_2}\cos\Theta_2 \text{ or} && (8)\\
\tan\Theta_2 &= \frac{1}{2}\tan\Theta_1 \text{ or} && (9)\\
\Theta_2 &= \arctan\left(\frac{1}{2}\tan\Theta_1\right) && (10)
\end{aligned}
$$

Inserting (10) into (6) and (7) gives the final answer for **question b.**, in addition to equ. (5):

$$
\begin{aligned}
T_2 &= \frac{mg}{\sin\arctan\frac{1}{2}\tan\Theta_1} && (11)\\
T_3 &= \frac{2mg}{\tan\Theta_1} && (12)
\end{aligned}
$$

To answer **question c.**, we note that D is equal to the sum of the horizontal projections of the 5 strings:

$$D = l + 2l\cos\Theta_1 + 2l\cos\Theta_2)$$

2 Problem 2

a. Approximating the orbit of the earth around the sun by a circle with $r = 1.5 \times 10^{11}$m, we find the magnitude of the velocity of the earth as the ratio of the distance covered in one revolution and the time interval for one revolution:

$$v = \frac{\Delta x}{\Delta t} = \frac{2\pi \times 1.5 \times 10^{11}\ \mathrm{m}}{365.25\ \mathrm{days}} = \frac{9.42 \times 10^{11}\ \mathrm{m}}{3.16 \times 10^{7}\ \mathrm{s}} = 3.0 \times 10^{4}\,\frac{\mathrm{m}}{\mathrm{s}}.$$

The direction of the velocity is perpendicular to the radius vector between sun and earth.

b. The magnitude of the centripetal acceleration is given by

$$a = \frac{v^2}{r} = \frac{(3.0 \times 10^4\mathrm{m/s})^2}{1.5 \times 10^{11}\mathrm{m}} = 0.006\frac{\mathrm{m}}{\mathrm{s}^2}.$$

The direction of the centripetal acceleration is along the radius vector, towards the sun.