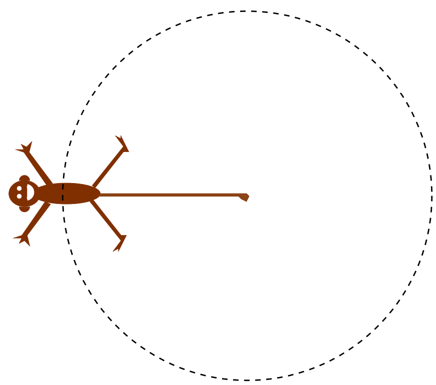
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X | Fall Term 2001

Solutions to PROBLEM SET 5

Problem 1 [10 pts]:

A child swings his stuffed monkey toy by the tail in a horizontal circle of radius 0.5 m. The monkey's mass is 0.5 kg. The tail is a bit chewed-on and frayed and can only withstand a tension of about 20 N.



a. What is the maximum speed with which the child can swing the monkey so that the tail does not break [5 pts]?

When the magnitude of the centripetal force is 20 N, the speed with which the child can swing the money is at a maximum. For circular motion, the centripetal force can be calculated as:

$$F_c = m\frac{v^2}{r} \qquad [3pts], \tag{1}$$

where $F_c = 20$ N. From Eqn. (1), we get $v = v_{max} = \sqrt{20} = 4.47$ m/s [2 pts].

b. What is the frequency ω corresponding to that speed [5 pts]?

$$T = \frac{2\pi r}{v}, \tag{2}$$

where T is the period and the frequency is $\frac{1}{T} = \frac{v}{2\pi r} = 1.42$ Hz. The angular velocity $\omega = \frac{2\pi}{T} = 8.9$ rad/s.

Problem 2 [10 pts]:

Design a carnival ride on which standing passengers are pressed against the inside curved wall of a rotating vertical cylinder. The passengers stand against the inside wall; the cylinder then starts rotating, and as it speeds up the passengers are pressed outward. When the cylinder is going fast enough the floor drops away leaving the passengers "suspended".

The cylinder is to turn at most at 0.5 rev per second. The maximum μ_s between clothing and wall is 0.2. What diameter should the cylinder have to safely allow the floor to drop away?

The floor can safely drop away from the cylinder when the static friction force is of the same magnitude as the gravitional force. It is pointing vertically up, opposite to the gravitional force and the net vertical force on the passenger is zero.

$$F_f = \mu_s N = \mu_s m\omega^2 r \qquad [4pts] \tag{3}$$

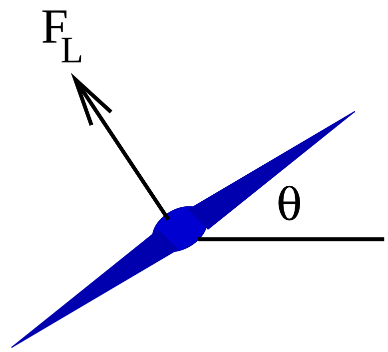
The normal force on the passenger arising from the contact between the passenger's clothing and the cylinder wall is just the centripetal force of the circular motion.

$$F_f = mg \qquad [4pts] \tag{4}$$

$\omega = \frac{2\pi}{T}$ and $T = 2$ sec. Thus, from Eqn. (3) and (4), we obtain $r = 5$ meters [2 pts].

Problem 3 [10 pts]:

Consider an airplane flying in a circle of diameter D. The circle is horizontal, i.e. the plane of the circle is parallel to the ground. The speed of the plane, v, is constant.



The pressure of the air against the bottom of the airplane's wings exerts a "lift" force F_L. The direction of this force is always perpendicular to the wings. The airplane has mass M. The airplane's engines exert a forward "thrust" force on the plane F_t, while air resistance exerts a backward "drag" force F_d. ("Forwards" and "backwards" mean with respect to the direction of motion of the airplane.)

a. Is F_t bigger, smaller, or the same magnitude as the drag force F_d? Explain [5 pts].

The magnitude of the thrust force F_t is the same as the magnitude of the drag force so that there is no net force in the tangential direction, the plane is in uniform circular motion in the horizontal plane.

b. At what angle θ must the airplane be banked at in order to continue flying in its circular path? Express your answer in terms of M, v and D [5 pts].

Decompose the lift force into two components: the vertical component ($F_l cos\theta$) and the horizontal component ($F_l sin\theta$). The net force in the vertical direction is zero and the net force in the horizontal direction provides the centripetal acceleration.

$$-Mg + F_l cos(\theta) = 0 \qquad [2pts] \tag{5}$$

$$F_l sin(\theta) = M\frac{v^2}{r} \qquad [2pts], \tag{6}$$

where $r = D/2$. From Eqn.(5) and (6), we obtain $\theta = tan^{-1}(\frac{v^2}{gD/2})$ [1 pt].