Solutions for 8.01x, homework 6 (Fall 2001)

1 Problem 1

The total weight of the two painters and the board is $(80 + 20 + 60)\text{kg} \times g = 1568\text{N}$. The total weight has to be supported by the two ropes

$$F_{T_1} + F_{T_2} = 1568\text{N} \tag{1}$$

In a static situation, the overall torque on the the board has to vanish. Picking the end of the board where rope 1 is attached as the pivot point and the origin of the coordinate system $x = 0$, we get the following condition:

$$\sum \tau = 0 = 784\text{N} \times 1\text{m} + 196\text{N} \times 2.5\text{m} + 588\text{N} \times x_B - F_{T_2} \times 5\text{m}, \tag{2}$$

where x_B is the position of painter B. Rearranging this equation, we find the following relation between the tension in the second rope and x_B:

$$F_{T_2} = 255\text{N} + 118\text{N} \times x_B/\text{m}. \tag{3}$$

Therefore F_{T_2} increases linearly with the position of painter B. The minimal F_{T_2} is 255N, the maximum for F_{T_2} for $x_B = 5\text{m}$ is 843N. Rope 2 will therefore not break.

By rearranging and inserting equ. (3) into equ. (1), we get the following equation for the tension in rope 1 (close to painter A):

$$F_{T_1} = 1568\text{N} - (255\text{N} + 118\text{N} \times x_B/\text{m}). \tag{4}$$

The tension in rope 1 decreases linearly with increasing x_B. It drops below the limit of 1000N for $x_B > 2.65\text{m}$. I.e. painter B has to be at least 2.65m from the end of the board to keep rope 1 (the one on painter A's side) from breaking.

2 Problem 2

a We find the force F_{musc} by requiring that the total torque on the spine vanishes. The pivot point is where the spine is supported by the lumbo-sacral disc. F_{disc} therefore doesn't contribute to the torque.

$$\sum \tau = 0 = F_{musc} \times \sin 12^o \times \frac{2}{3}L - w_{trunk} \times \frac{1}{2}L \times \cos 35^o - w_{head,arms} \times L \times \cos 35^o \tag{5}$$

Solving for F_{musc} and inserting the numbers for w_{trunk} and $w_{head,arms}$ this yields

$$F_{musc} = (0.4 \times 60\text{kg} \times 9.8\frac{m}{s^2} \times \frac{1}{2}L + 0.260\text{kg} \times 9.8\frac{m}{s^2} \times L) \cos 35^o \times \frac{3}{2L \times \sin 12^o}, \tag{6}$$

giving $F_{musc} = 1390\text{N}$.

b To find the magnitude of F_{disc}, we sum up the components of F_{musc}, w_{trunk} and $w_{head,arms}$. The angle between F_{musc} and the horizontal is $35^o - 12^o = 23^o$.

$$\sum F_x = 0 = F_{disc_x} - F_{musc}\cos 23^o \tag{7}$$
$$\sum F_y = 0 = F_{disc_y} - F_{musc}\sin 23^o - w_{trunk} - w_{head,arms} \tag{8}$$

This gives $F_{disc_x} = 1280$N and $F_{disc_y} = 896$N. The magnitude of F_{disc} is $\underline{F_{disc} = \sqrt{F^2_{disc_x} + F^2_{disc_y}} = 1562 \text{ N}}$. The angle β of F_{disc} relative to the horizontal is $\underline{\arctan F_{disc_y}/F_{disc_x} = 35^o}$.

c The force F_{disc} corresponds to around <u>2.5 times the body weight</u>.

d As the angles β and Θ are identical, the compressive force acting along the normal vector of the disc is <u>equal to F_{disc}, but in opposite direction</u>.

e When lifting the weight, $w_{head,arms}$ is doubled. Following the same calculation as above, F_{muscle} increases to 2085N and the magnitude of F_{disc} increases to 2310N. The resulting angle β is 33.8^o. <u>The normal force on the disc is 2309N</u>.

f As equ. (6) shows, the force exerted by the muscle is proportional to $\cos\Theta$. Therefore, one should keep the spine vertical ($\Theta = 90^o$), such that F_{muscle} vanishes and F_{disc} is just given by $w_{head,arms} + w_{trunk} +$ the weight to be lifted.