Solutions for 8.01x, homework 6 (Fall 2001)

1 Problem 1

The total weight of the two painters and the board is \((80 + 20 + 60)\text{kg} \times g = 1568\text{N}\). The total weight has to be supported by the two ropes

\[
F_{T_1} + F_{T_2} = 1568\text{N}
\]  

(1)

In a static situation, the overall torque on the the board has to vanish. Picking the end of the board where rope 1 is attached as the pivot point and the origin of the coordinate system \(x = 0\), we get the following condition:

\[
\sum \tau = 0 = 784\text{N} \times 1\text{m} + 196\text{N} \times 2.5\text{m} + 588\text{N} \times x_B - F_{T_2} \times 5\text{m},
\]  

(2)

where \(x_B\) is the position of painter B. Rearranging this equation, we find the following relation between the tension in the second rope and \(x_B\):

\[
F_{T_2} = 255\text{N} + 118\text{N} \times x_B/\text{m}.
\]  

(3)

Therefore \(F_{T_2}\) increases linearly with the position of painter B. The minimal \(F_{T_2}\) is 255N, the maximum for \(F_{T_2}\) for \(x_B = 5\text{m}\) is 843N. Rope 2 will therefore not break.

By rearranging and inserting equ. (3) into equ. (1), we get the following equation for the tension in rope 1 (close to painter A):

\[
F_{T_1} = 1568\text{N} - (255\text{N} + 118\text{N} \times x_B/\text{m}).
\]  

(4)

The tension in rope 1 decreases linearly with increasing \(x_B\). It drops below the limit of 1000N for \(x_B > 2.65\text{m}\). I.e. painter B has to be at least 2.65m from the end of the board to keep rope 1 (the one on painter A’s side) from breaking.

2 Problem 2

a  We find the force \(F_{\text{muscl}}\) by requiring that the total torque on the spine vanishes. The pivot point is where the spine is supported by the lumbo-sacral disc. \(F_{\text{disc}}\) therefore doesn’t contribute to the torque.

\[
\sum \tau = 0 = F_{\text{muscl}} \times \sin 12^\circ \times \frac{2}{3}L - w_{\text{trunk}} \times \frac{1}{2}L \times \cos 35^\circ - w_{\text{head,arms}} \times L \times \cos 35^\circ
\]  

(5)

Solving for \(F_{\text{muscl}}\) and inserting the numbers for \(w_{\text{trunk}}\) and \(w_{\text{head,arms}}\) this yields

\[
F_{\text{muscl}} = (0.4 \times 60\text{kg} \times 9.8\text{m/s}^2 \times \frac{1}{2}L + 0.260\text{kg} \times 9.8\text{m/s}^2 \times L) \cos 35^\circ \times \frac{3}{2L \times \sin 12^\circ},
\]  

(6)

giving \(F_{\text{muscl}} = 1390\text{N}\).
b To find the magnitude of \( F_{\text{disc}} \), we sum up the components of \( F_{\text{musc}}, w_{\text{trunk}} \) and \( w_{\text{head,arms}} \.
The angle between \( F_{\text{musc}} \) and the horizontal is \( 35^\circ - 12^\circ = 23^\circ \).

\[
\begin{align*}
\sum F_x &= 0 = F_{\text{disc}_x} - F_{\text{musc}} \cos 23^\circ \\
\sum F_y &= 0 = F_{\text{disc}_y} - F_{\text{musc}} \sin 23^\circ - w_{\text{trunk}} - w_{\text{head,arms}}
\end{align*}
\]  

(7)

(8)

This gives \( F_{\text{disc}_x} = 1280\)N and \( F_{\text{disc}_y} = 896\)N. The magnitude of \( F_{\text{disc}} \) is \( F_{\text{disc}} = \sqrt{F_{\text{disc}_x}^2 + F_{\text{disc}_y}^2} = 1562\) N.
The angle \( \beta \) of \( F_{\text{disc}} \) relative to the horizontal is \( \arctan \frac{F_{\text{disc}_y}}{F_{\text{disc}_x}} = 35^\circ \).

c The force \( F_{\text{disc}} \) corresponds to around 2.5 times the body weight.

d As the angles \( \beta \) and \( \Theta \) are identical, the compressive force acting along the normal vector of the disc is equal to \( F_{\text{disc}} \), but in opposite direction.

e When lifting the weight, \( w_{\text{head,arms}} \) is doubled. Following the same calculation as above, \( F_{\text{musc}} \) increases to 2085N and the magnitude of \( F_{\text{disc}} \) increases to 2310N. The resulting angle \( \beta \) is \( 33.8^\circ \). The normal force on the disc is 2309N.

f As equ. (6) shows, the force exerted by the muscle is proportional to \( \cos \Theta \). Therefore, one should keep the spine vertical (\( \Theta = 90^\circ \)), such that \( F_{\text{musc}} \) vanishes and \( F_{\text{disc}} \) is just given by \( w_{\text{head,arms}} + w_{\text{trunk}} + \) the weight to be lifted.