MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X Fall Term 2001

50 points total

Problem 1 (10 points)

a) (4 points) Consider a typical bowling ball with a mass (*m*) of 5 Kg and dropped from a height (*h*) of 1 meter. The potential energy stored in bowling ball is *mgh*. Using 10 m/s^2 for *g*, the potential energy is 50 Joules.(3 points - anything between 10 Joules and 200 Joules is acceptable here) When the ball falls onto your foot, this energy goes into work done by stretching the ligaments of your foot (and otherwise rearranging your foot) and into creating vibrations in your foot and the ground.(1 point)

b) (6 points) Imagine that the truck is moving 40 meters/second (about 88 miles per hour). To determine the required elevation rise of the ramp, we set:

$$\frac{1}{2}mv^2 = mgh \text{ which gives } h = \frac{v^2}{2g}. \quad \text{(4 points)}$$

Two possible ways to solve this problem:

1) no friction between truck and road:

Notice that mass of the truck drops out of this equation. Plugging in the numbers, we find that the elevation rise of the ramp should be about 80 meters (1 point any number between 30 and 300 meters is acceptable here). The truck can't climb a straight wall – its link to its trailer might break if we tilt the ramp too steeply. On the other hand, we don't want to spend a huge amount of money making a ramp that is too long. So, let's take an angle of 15 degrees (ranges 5 to 45 degrees acceptable). This makes the length of the ramp, $l = h/\sin\theta$, about 310 meters.

2) With a coefficient of friction:

You can also solve this problem by adding a coefficient of friction between the truck and the road. Above, we've solved it for the case of zero friction. Using friction will obviously shorten the distance required to stop the truck. Solving the problem this way requires the use of the energy balance equation including the energy dissipated in friction with the road.

$$\frac{1}{2}mv^2 = mgl\sin\theta + \mu_k mgl\cos\theta .$$

Problem 2 (20 points)

a) (6 points) Take l as the distance between the starting point of the mass and the position at which it first touches the spring. θ is the slope of the ramp. Take x as the distance the spring is compressed. Considering all of the gravitational potential energy of the mass converted to potential energy of the compressed spring, we get:

$$mg(l+x)\sin\theta = \frac{1}{2}kx^2. \text{ (3 points)}$$

Solving this equation for x, *we find:*

$$x = \frac{mg}{k}\sin\theta \pm \sqrt{\left(\frac{mg}{k}\sin\theta\right)^2 + 2\frac{mg}{k}l\sin\theta} \text{ (1 point)}$$

The + solution corresponds to maximum compression of the spring (1 point) Plugging in numbers for m, k ,g, and θ, we get x=0.99 meters.

b) (7 points) If the slope has a surface with friction, the some of the potential energy of the mass is dissipated as work done against the force of friction. The force of friction here has a magnitude of $\mu_k mg\cos\theta$. Therefore, the energy balance equation becomes: $mg(l+x)\sin\theta - \mu_k mg\cos\theta(l+x) = \frac{1}{2}kx^2$. (3 points)

Solving this, we find:

$$x = \frac{mg}{k}(\sin\theta - \mu_k\cos\theta) \pm \sqrt{\left(-\frac{mg}{k}(\sin\theta - \mu_k\cos\theta)\right)^2 + 2\frac{mg}{k}l(\sin\theta - \mu_k\cos\theta)}$$

(2 points)

Using the + solution, plugging in the numbers, and using μ_k=0.2, we find x=0.78 meters. (1 point)

c) (5 points) Now the energy in the compressed spring (at position x_{comp}) is converted back into potential energy of the mass and to energy of the extended spring (at position x_{ext}). The spring also does work against friction. The energy balance equation is:

$$\frac{1}{2}kx_{comp}^2 = mg(x_{comp} - x_{ext})\sin\theta + \frac{1}{2}kx_{ext}^2 + \mu_k mg\cos\theta(x_{comp} - x_{ext})$$

rearranging terms:

$$\frac{1}{2}kx_{ext}^2 - mg(\sin\theta + \mu_k\cos\theta)x_{ext} + mg(\sin\theta + \mu_k\cos\theta)x_{comp} - \frac{1}{2}kx_{comp}^2 = 0$$

Solving this quadratic equation for x_{ext}, we get:

$$x_{ext} = \frac{mg}{k}(\sin\theta + \mu_k \cos\theta) \pm \sqrt{\left(\frac{mg}{k}(\sin\theta + \mu_k \cos\theta)\right)^2 - 2\frac{mg}{k} x_{comp} (\sin\theta + \mu_k \cos\theta) + x_{comp}^2}$$

We use the minus solution to obtain the maximum extension of the spring x_{ext} being negative means that the spring is stretched from its rest position. Plugging in the numbers and using x_{comp}=0.78 meters, this gives x_{ext}=-0.52 meters.

d) After being fully compressed, the spring pushes back against the mass, causing it to overshoot the equilibrium position of the mass and the spring, and the mass oscillates back and forth about the equilibrium position. The amplitude of this oscillation decreases with time until the net force on the mass is no longer large enough to overcome friction. (2 points)

Problem 3 (10 points)

a) (4 points) Considering the actor as simply hanging (stationary) from the cable, the net upward force exerted by the cable must be equal to the actor's weight (*mg*). Therefore:

$$T\cos\theta = mg \text{ or } T = \frac{mg}{\cos\theta}$$

This tension cannot exceed the weight of the sandbag (*Mg*). So:

$$Mg < \frac{mg}{\cos\theta} \text{ or } \cos\theta < \frac{m}{M}$$

Plugging in the numbers for *M* and *m*, this gives θ_{max}=60 degrees.

b) (6 points) The actor starts at a height $R(1\text{-}\cos\theta)$ above the floor, and the cable is vertical after the swoop. The energy balance equation gives the actor's velocity upon reaching the floor:

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2 \text{ or } v^2 = 2gR(1-\cos\theta).$$

At the bottom of the swing, the tension in the rope must both balance the actor's weight and provide sufficient additional force to keep him swinging in a circle of radius *R*. The force equation gives is then:

$$T = mg + m\frac{v^2}{R} = mg\left(1 + \frac{2R(1-\cos\theta)}{R}\right) = mg(3 - 2\cos\theta).$$

Plugging in the numbers, we get *T*=1090 Newtons.

Problem 4 (10 points)

a) (5 points) The escape speed is found by setting the sum of the gravitational potential energy (negative) and the object's kinetic energy equal to zero. This gives:

$$-\frac{GMm}{r}+\frac{1}{2}mv^2=0 \text{ or } v=\sqrt{\frac{2GM}{r}}.$$

Here, M is the mass of Mercury, G is the gravitational constant (6.67 x 10^{-11} $m^3\ kg^{-1}\ s^{-2}$), and r is the radius of the planet. This gives v_{escape}=4170 m/s.

b) (5 points) Here, we have the energy balance equation:

$$-\frac{GMm}{r}+\frac{1}{2}mv_{initial}^2=\frac{1}{2}mv_{final}^2 \text{ or } v_{initial}=\sqrt{\frac{2GM}{r}+v_{final}^2}$$

Plugging in v_{final}=50,000 m/s, we get $v_{initial}$=50200 m/s.