8.01x Problem Set 8 (11/2/01)

1 Problem 1

a The total power developed by the water is given by the potential energy per unit time that's transferred into kinetic energy. Potential energy for the mass flowing across the fall in one second is:

$$E_{pot} = m \cdot g \cdot h, \tag{1}$$

with $h = 108$ m, $g = 9.8$ m/s^2 and $m = 1.58 \cdot 10^6$ kg. The resulting power per second is

$$P = E_{pot}/\Delta t = 1.67 \cdot 10^9\text{W} = 1.67 GW. \tag{2}$$

This is comparable to a nuclear powerstation.

b The kinetic energy of the spacecraft is

$$E_{kin} = 1/2 \cdot m \cdot v^2 = 8.18 \cdot 10^7\text{Nm}. \tag{3}$$

Using 150 kW of power, it will take

$$\Delta t = E_{kin}/P = 545\text{s} \tag{4}$$

to accelerate the spacecraft to a velocity of 600 mph.

2 Problem 2

The total initial energy of the car is

$$E_{tot} = 1/2 \cdot m \cdot v^2 + m \cdot g \cdot h = 520\text{kJ}, \tag{5}$$

where $h = 30\text{m} \cdot \sin 30^o$ is the difference in height of the car before and after braking. $m \cdot g \cdot h$ therefore is the difference in potential energy before and after braking. The total inital energy is converted into heat. Half of the heat is absorbed by the tires. Each tire therefore absorbs 65 kJ. That corresponds to an increase in temperature of

$$\Delta T = 65\text{ kJ} \cdot 4 \cdot 10^{-5}\text{K/J} = 2.6\text{K}. \tag{6}$$

3 Problem 3

a The equation of motion for the mass of the spring is

$$x(t) = A\cos(\omega t + \phi) \tag{7}$$

We know that $\omega = \sqrt{k/m}$. To determine A, we compare the potential energy at the extreme of the motion (*i.e.* $x = A$), with the total initial energy.

$$E_{tot} = 1/2 \cdot k \cdot D^2 + 1/2 \cdot m \cdot v_0^2 = 1/2 \cdot k \cdot A^2 \tag{8}$$

Therefore

$$A = \sqrt{(m/k)v_0^2 + D^2} \tag{9}$$

We find ϕ by looking at the equation of motion for $t = 0$ (i.e. $x = D$):

$$D = A\cos(\omega t + \phi) = \sqrt{(m/k)v_0^2 + D^2}\cos\phi \tag{10}$$

This gives

$$\phi = \arccos(D/\sqrt{(m/k)v_0^2 + D^2}) \tag{11}$$

and the following equation of motion:

$$x(t) = \sqrt{(m/k)v_0^2 + D^2}\cos(\sqrt{k/m}t + \arccos D/\sqrt{(m/k)v_0^2 + D^2}) \tag{12}$$

The velocity as a function of t is therefore

$$v(t) = \frac{dx(t)}{dt} = -\sqrt{v_0^2 + D^2 \cdot \omega^2}\sin(\omega t + \arccos D/\sqrt{(m/k)v_0^2 + D^2} \tag{13}$$

b The block is motionless when $x = \pm A$. The force at this point is $F = -k \cdot A$. This gives an acceleration at A of

$$a = F/m = -k/m\sqrt{(m/k)v_0^2 + D^2}. \tag{14}$$

c The velocity v_{max} at $x = 0$ can be obtained from conservation of energy:

$$1/2 \cdot m \cdot v_{max}^2 = 1/2 \cdot m \cdot v_0^2 + 1/2 \cdot k \cdot D^2. \tag{15}$$

Therefore

$$v_{max} = \sqrt{v_0^2 + (k/m)D^2}. \tag{16}$$

4 Problem 4

We solve the problem using conservation of energy.

a The initial potential energy (due to the compression of the spring) is $E(t=0) = 1/2 \cdot k \cdot d^2$. The energy at take-off is giving by the kinetic energy and the gravitational potential energy of the block. The height of the block when it is launched is just r:

$$E_{takeoff} = 1/2 \cdot m \cdot v^2 + m \cdot g \cdot r = 1/2 \cdot k \cdot d^2. \tag{17}$$

The speed is therefore

$$v = \sqrt{((k/m) \cdot d^2 - 2 \cdot g \cdot r)} \tag{18}$$

b It reaches maxium height h_{max} when all initial energy has been converted to gravitational potential energy:

$$m \cdot g \cdot h = 1/2 \cdot k \cdot d^2 \text{ or} \tag{19}$$

$$h = 1/2 \cdot k \cdot d^2/(m \cdot g) \tag{20}$$