Massachusetts Institute of Technology
Physics Department

8.01X — Fall 2002

EXPERIMENT AM; ANGULAR MOMENTUM

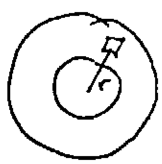
DATA ANALYSIS

Moment of Inertia:

a) In experiment AM, A 1" US Standard Washer has inner radius r_i and an outer radius r_o. **Calculate the moment of inertia I_1 and I_2 of each washer from the expression,**

$$I_1 = \frac{1}{2} M_1 (r_i^2 + r_0^2)$$

where M_1 is the mass of washer #1 and where r_i and r_o are the inner and outer radii respectively.



$$I = \int dm\, r_\perp^2 = \int_{r=r_i}^{r=r_o} \int_{\theta=0}^{2\pi} (\sigma\, r\, dr\, d\theta)\, r^2$$

where $\sigma = \frac{m}{\text{Area}} = \frac{m}{\pi(r_o^2 - r_i^2)}$. The integral becomes

$$I = \frac{2\pi m}{\pi(r_o^2 - r_i^2)} \int_{r_i}^{r_o} r^3 dr = \frac{2m}{r_o^2 - r_i^2} \left. \frac{r^4}{4} \right|_{r_i}^{r_o} = \frac{1}{2} \frac{m(r_o^4 - r_i^4)}{r_o^2 - r_i^2}$$

$$I = \frac{1}{2} \frac{m(r_o^2 + r_i^2)(r_o^2 - r_i^2)}{r_o^2 - r_i^2} = \frac{1}{2} m (r_o^2 + r_i^2).$$

My washer #1+2 had $r_i = 13.8$ mm, $r_o = 31.5$ mm.

$m_1 = 83 \times 10^{-3}$ kg, $m_2 = 92 \times 10^{-3}$ kg. Thus

$$I_1 = \frac{1}{2}(83 \times 10^{-3} \text{ kg})\left((31.5 \times 10^{-3} \text{ m})^2 + (13.8 \times 10^{-3} \text{ m})^2\right) = 4.91 \times 10^{-5} \text{ kg-m}^2$$

$$I_2 = \frac{1}{2}(92 \times 10^{-3} \text{ kg})\left((31.5 \times 10^{-3} \text{ m})^2 + (13.8 \times 10^{-3} \text{ m})^2\right) = 5.44 \times 10^{-5} \text{ kg-m}^2$$

Calibration of the Motor/Generator:

b) The voltage generated by the motor is linearly proportional to the angular velocity of the motor, $\omega = \beta V$, where β is the constant of proportionality. Recall that the angular frequency $\omega = 2\pi f$; where f is the frequency. Do not confuse the two quantities or their units: ω (rad/s) and f (Hz). **Calculate your value for β from your stroboscopic measurement of voltage generated by the motor when it is spinning at 30 Hz.**

I found that 1.40 ± 0.5 V = 30 Hz. Thus $\beta = 2\pi\,(30\text{ Hz}) / 1.40\text{ V} = 1.35 \times 10^{2}$ rad/sec−V.

Experiment 1:

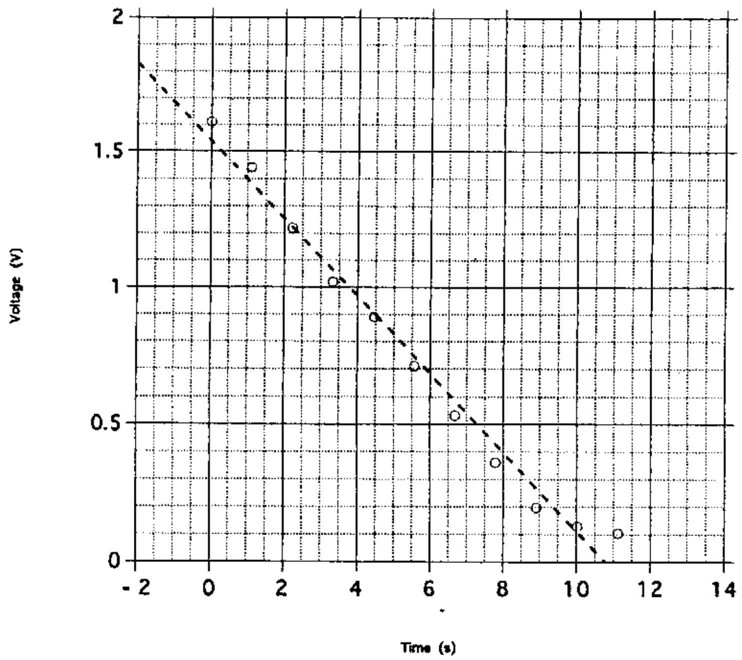
Plot the voltage readings versus time for the three runs on three sheet of linear graph paper, arranging to have the first reading always at the zero of time. Select the most representative run and draw the best-fit straight line through the combined points. Use the slope of the line to deduce the frictional torque that slows the washer. Here's how to do that. Convert the measured slope of your best-fit line, dV/dt to a value for $d\omega/dt$ using the calibration you made for voltage and angular frequency. Recall that the angular acceleration is $\alpha = d\omega/dt$ hence the frictional torque is given by $\tau = I_1\alpha$. Use SI units throughout; do not mix units from different systems.

- "Data expt 1 11/21/99"

	time (sec)	expt1.1 V	expt 1.2 V	expt 1.3 V	expt 1.4 V	expt 1.5 V
0	0.00	1.61	1.64	1.66	1.59	1.65
1	1.11	1.44	1.39	1.40	1.35	1.39
2	2.23	1.22	1.18	1.18	1.21	1.17
3	3.34	1.02	0.990	0.990	1.01	1.04
4	4.45	0.890	0.810	0.810	0.820	0.850
5	5.57	0.710	0.690	0.690	0.650	0.680
6	6.68	0.530	0.520	0.520	0.530	0.500
7	7.79	0.360	0.350	0.340	0.360	0.340
8	8.91	0.198	0.207	0.189	0.129	0.184
9	10.0	0.129	0.128	0.106		0.110
10	11.1	0.104	0.0140			

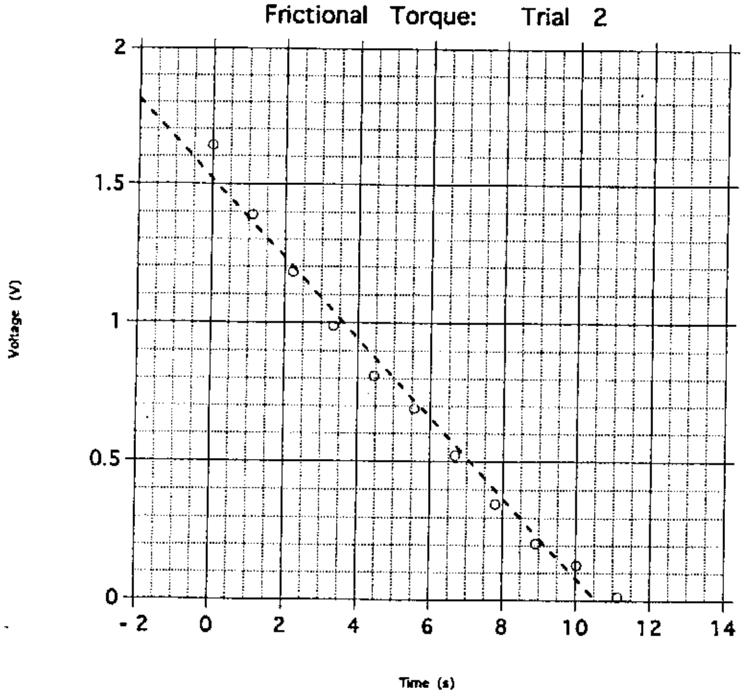
-----y = 1.54 - 0.143x R= 0.992

Frictional Torque: Trial 1



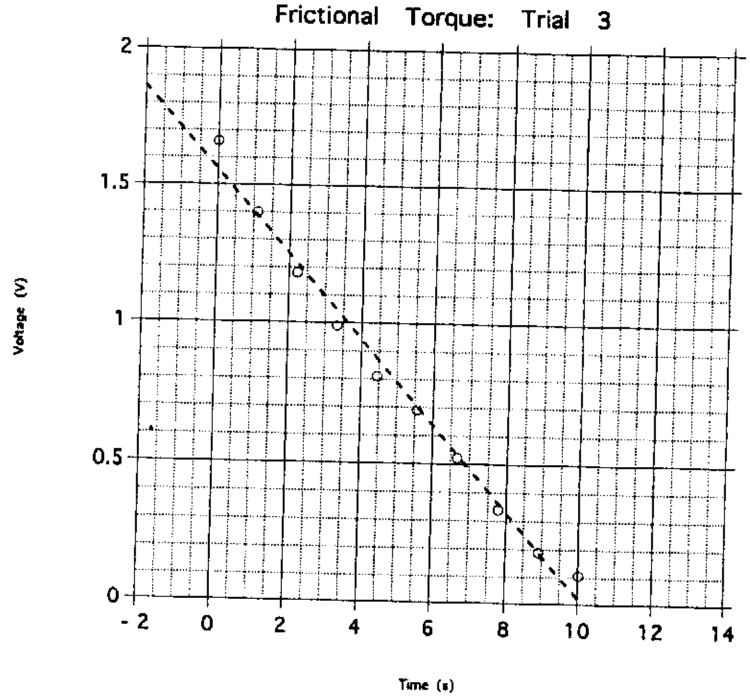
-----y = 1.52 - 0.144x R= 0.993

Frictional Torque: Trial 2



-----y = 1.56 - 0.153x R= 0.994

Frictional Torque: Trial 3



Trial 4

- "data frictional torque"

	Time (s)	Voltage (v)
0	0.00	1.59
1	1.11	1.35
2	2.23	1.21
3	3.34	1.01
4	4.45	0.820
5	5.57	0.650
6	6.68	0.530
7	7.79	0.360
8	8.91	0.129

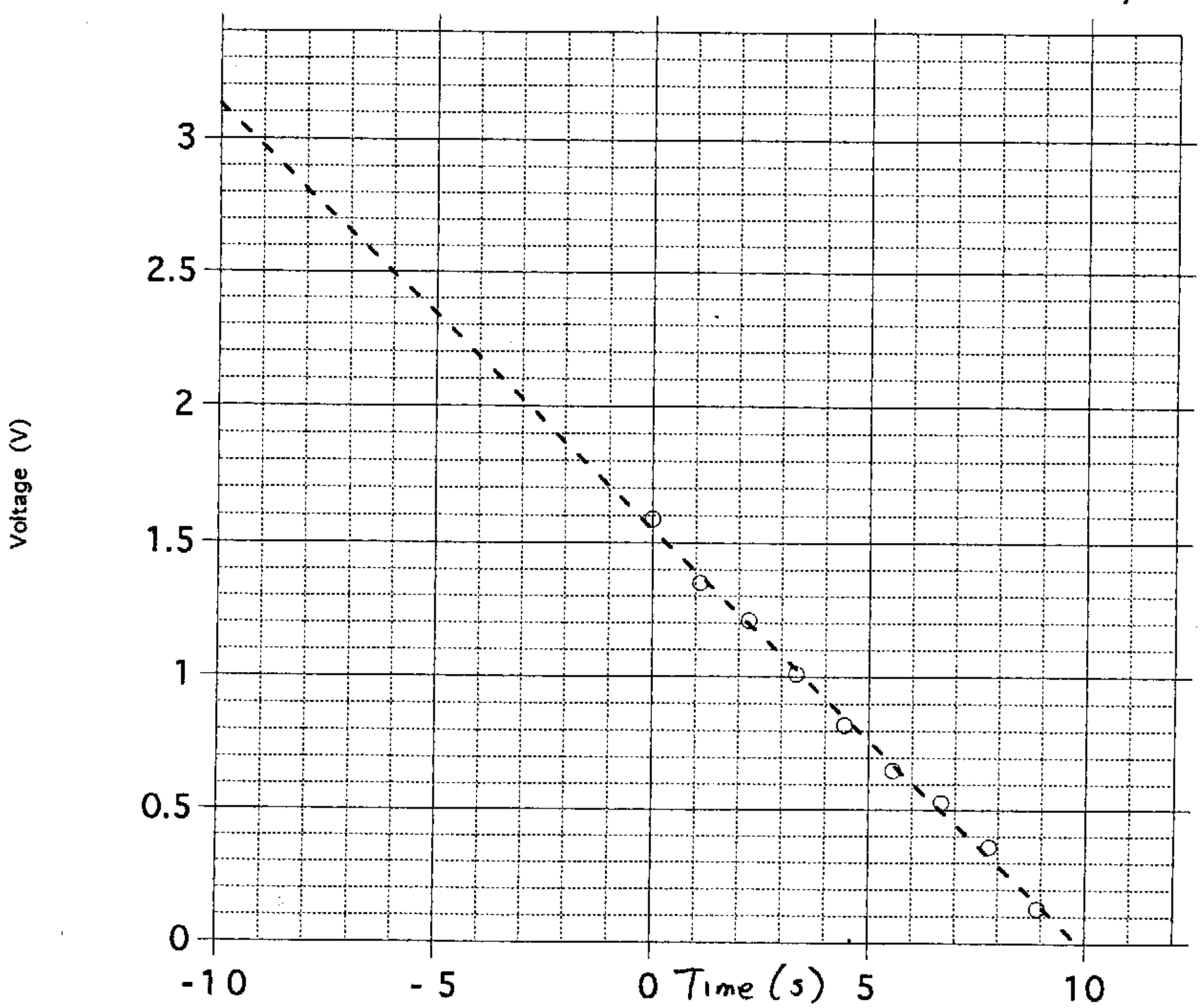
--- y = 1.55 - 0.158x R= 0.998

The slope $= \frac{\Delta V}{\Delta t} = .158 \frac{V}{s}$. The angular acceleration

$$\alpha = \frac{\Delta \omega}{\Delta t} = \beta \frac{\Delta V}{\Delta t} = (1.35 \times 10^{2} \frac{rad}{sec\text{-}V})(.158 \frac{V}{s})$$

$$\alpha = 2.13 \times 10^{1}\ rad/s^2.$$

Frictional Torque: One Washer I_1 Trial 4



c) Calculate the frictional torque of the motor for Expt. 1.

The frictional torque

$$\tau = I_1 \alpha = (4.91 \times 10^{-5}\ kg\text{-}m^2)(2.13 \times 10^{1} \frac{rad}{s^2}) = 1.0 \times 10^{-3}\ J$$

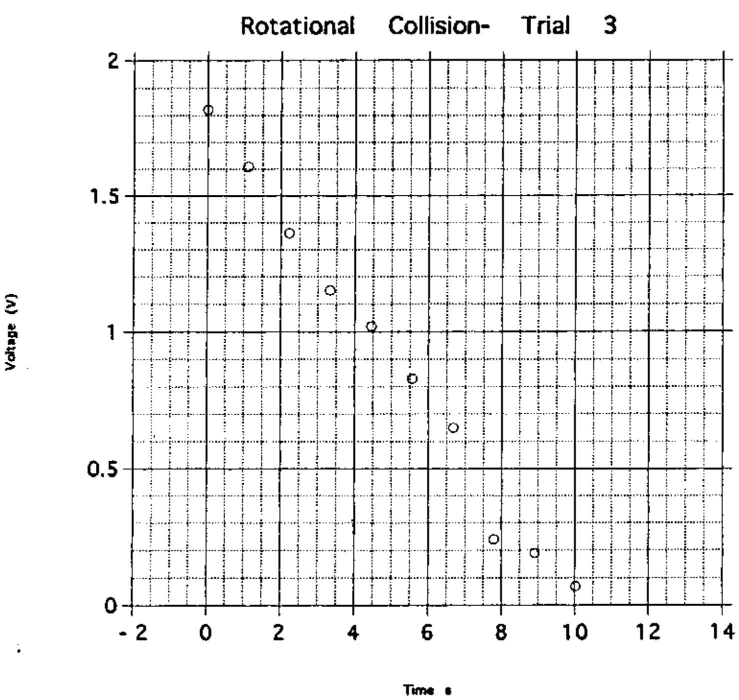
Experiment 2.

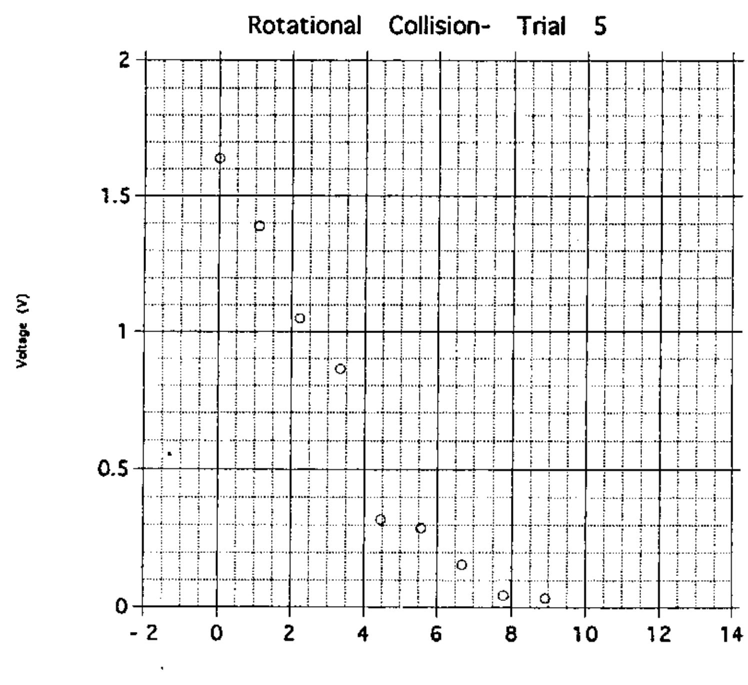
Here we will calculate the angular momentum just before and after the inelastic rotational collision as a check on the near conservation of angular momentum.

Plot the voltage readings versus time for each of the three runs;

"Rotational Collision- Data "

	time s	expt 2.1 V	expt 2.2 V	expt 2.3 V	expt 2.4 V	expt 2.5 V
0	0.00	1.59	1.76	1.82	1.60	1.64
1	1.11	1.43	1.57	1.61	1.36	1.39
2	2.23	1.22	1.33	1.36	1.16	1.05
3	3.34	1.02	1.12	1.15	0.830	0.860
4	4.45	0.580	0.990	1.02	0.390	0.320
5	5.57	0.350	0.740	0.830	0.217	0.289
6	6.68	0.165	0.350	0.650	0.208	0.156
7	7.79		0.280	0.240	0.134	0.0430
8	8.91		0.203	0.189	0.0270	0.0340
9	10.0		0.142	0.0690		
10	11.1					
11	12.2					





Trial 2

- "data two washers #4"

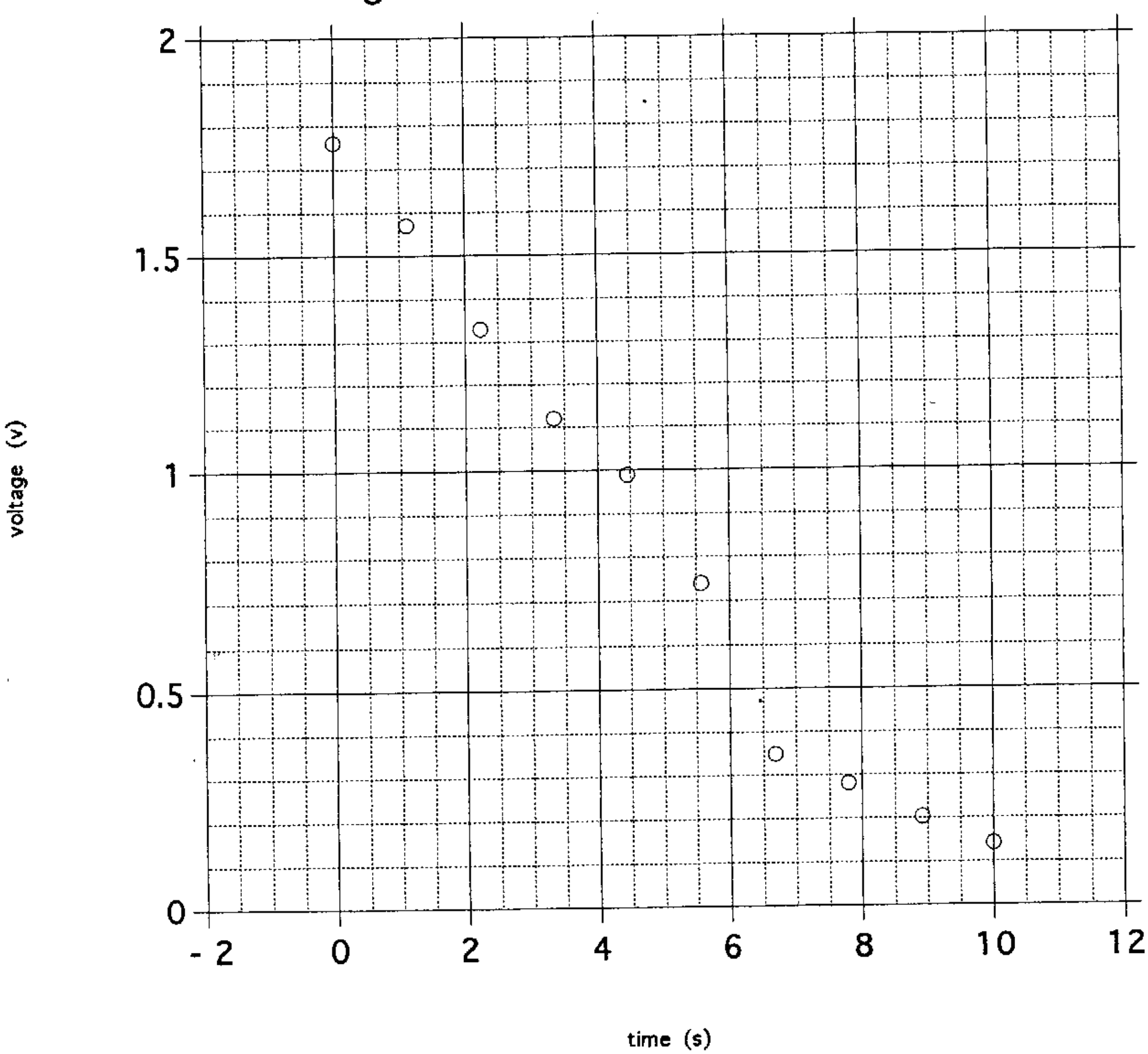
	time (s)	voltage (v)
0	0.00	1.76
1	1.11	1.57
2	2.23	1.33
3	3.34	1.12
4	4.45	0.990
5	5.57	0.740
6	6.68	0.350
7	7.79	0.280
8	8.91	0.203
9	10.0	0.142

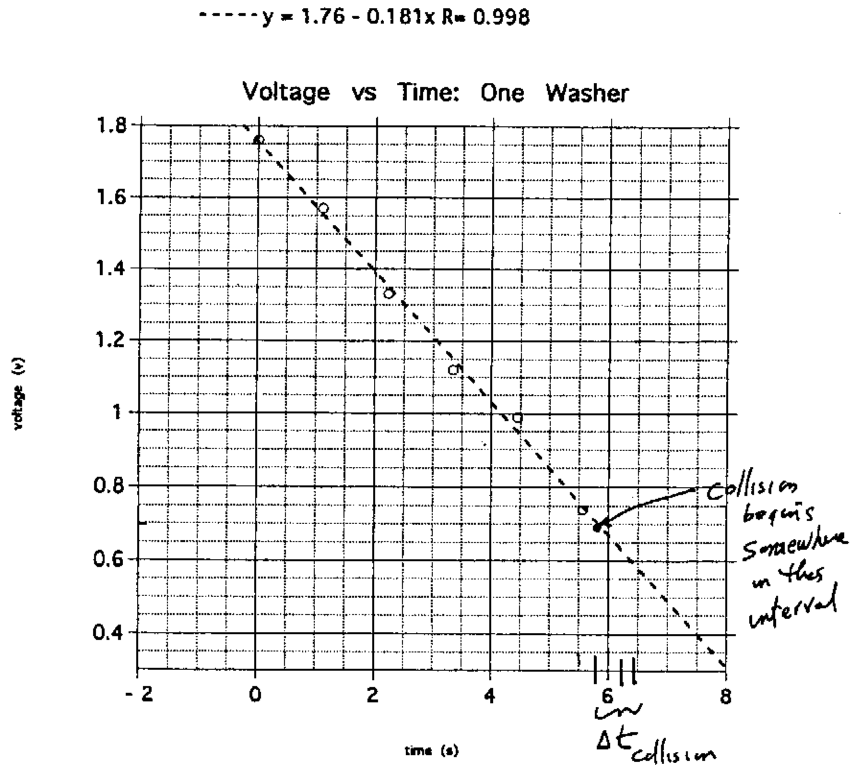
error in data taking PD
adjusted value

$y = 1.76 - 0.181x$ R= 0.998 $\boxed{0 < t < 6\text{ s}}$

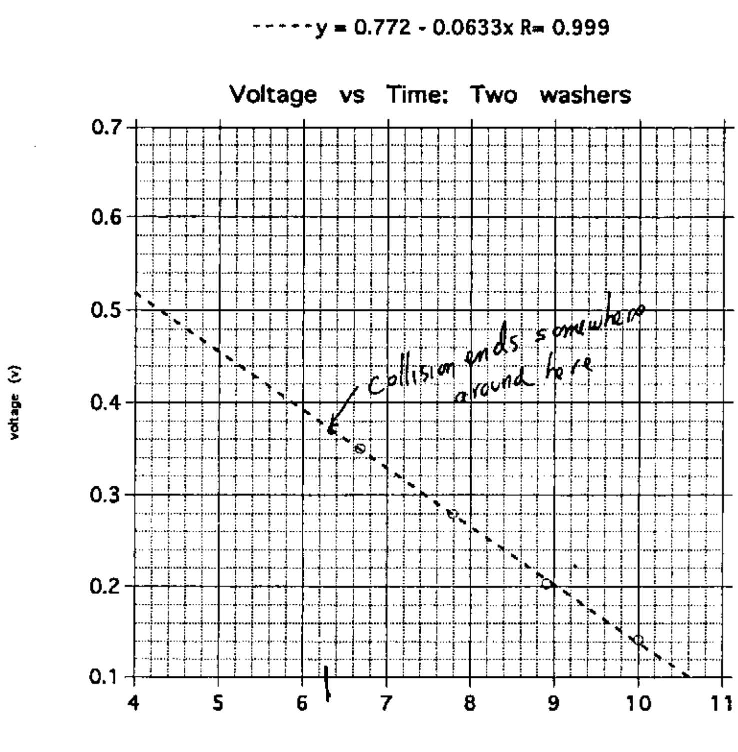
$y = 0.772 - 0.0633x$ R= 0.999 $\boxed{6\text{ s} < t < 12\text{ s}}$

Voltage vs Time: Rotational Collision Trial 2





-----y = 1.76 - 0.181x R= 0.998
Voltage vs Time: One Washer
voltage (v)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
-2
0
2
4
6
8
time (s)
collision begins somewhere in this interval
Δt collision



-----y = 0.772 - 0.0633x R= 0.999
Voltage vs Time: Two washers
voltage (v)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
4
5
6
7
8
9
10
11
time (s)
collision ends somewhere around here

Select the most representative graph; the graph should have at least three or four data points for the slowing down of the first washer, and three or four data points for the slowing down of the two washers. The collision time is on the order of 0.5 seconds so it is unlikely that you have more than at most one data point for the collision.

Draw a 'best-fit' straight line through the data points preceding the time the washer was dropped. Draw a second bet-fit straight line for the data points after the collision. Extend both lines into the time where the collision occurred. Try to estimate when the collision occurred. Calculate the voltage just before the collision occurred using your first straight line. Then calculate the voltage 0.5 seconds after the collision occurred using your second straight line. Again convert your voltages to angular frequencies using your calibration.

The two data points on either side of the collision occured at $t = 5.57\,s$ and $t = 6.68\,s$. Lets assume these mark the beginning and end of the collision.

$V_{before} = 1.76 - (0.181)(5.57\,s) = .75$ Volts

$W_{before} = \beta\, V_{before} = \left(1.35 \times 10^2 \frac{rad}{sec-V}\right)(.75\,\text{Volts}) = 1.0 \times 10^2 \frac{rad}{sec}$

$V_{after} = .77 - (0.063)(6.68\,s) = .35$ Volts

$W_{after} = \beta\, V_{after} = \left(1.35 \times 10^2 \frac{rad}{sec-V}\right)(.35\,\text{Volts}) = 4.7 \times 10^1 \frac{rad}{sec}$

d) **Calculate the frictional torque of the motor for both of your best-fit straight lines. Recall that for your data after the collision $\tau = (I_1 + I_2)\alpha_a$. Does the frictional torque of the motor change when there are two washers on it?** Similar to the calculation is part c): $\alpha_1 = \beta\left(\frac{\Delta V}{\Delta t}\right)_1 = -(1.35\times10^2 \frac{rad}{V\text{-}sec})(.181 \frac{V}{s})$

$\alpha_1 = 2.4\times10^1\ rad/s^2$; $\tau_1 = I_1\alpha_1 = (4.9\times10^{-5}\ kg\text{-}m^2)(2.4\times10^1 \frac{rad}{s^2})$

$\tau_1 = 1.2\times10^{-3}\ J$. When both washers were on:

$\alpha_2 = \beta\left(\frac{\Delta V}{\Delta t}\right)_2 = (1.35\times10^2 \frac{rad}{V\text{-}sec})(0.63 \frac{V}{sec}) = 8.5\ rad/s^2.$

$\tau_2 = (I_1+I_2)\alpha_2 = (4.91\times10^{-5}\ kg\text{-}m^2 + 5.44\times10^{-5}\ kg\text{-}m^2)(8.5\ rad/s^2)$

$\tau_2 = 8.8\times10^{-4}\ J$. The frictional torque does change

$\tau_{ave} = (\tau_1+\tau_2)/2 = 1.0\times10^{-3}\ J$. This agrees with the calculation in part c).

e) Compute the angular momentum $L_b = I_1\omega_b$, (where I_1 is the moment of inertia of washer #1) of the single washer just before the inelastic rotational collision. Use SI units.

$L_b = I_1\omega_{before} = (4.9\times10^{-5}\ kg\text{-}m^2)(1.0\times10^2 \frac{rad}{sec}) = 5.0\times10^{-3}\ kg\frac{m^2}{s}$

f) Calculate the angular momentum of the two-washer system just after the collision, $L_a = (I_1 + I_2)\omega_a$, where I_2 is the moment of inertia of washer #2.

$L_{after} = (I_1+I_2)\omega_{after} = (4.91\times10^{-5}\ kg\text{-}m^2 + 5.44\times10^{-5}\ kg\text{-}m^2)(4.7\times10^1 \frac{rad}{sec})$

$L_{after} = 4.9\times10^{-3}\ kg\text{-}\frac{m^2}{s}$.

g) Calculate the change $\Delta L \equiv L_a - L_b$ in angular momentum during the rotational collision.

$\Delta L = 4.9\times10^{-3}\ kg\text{-}\frac{m^2}{s} - 5.0\times10^{-3}\ kg\frac{m^2}{s} = -0.1\times10^{-3}\ kg\text{-}\frac{m^2}{s}$

h) Calculate the fractional change in angular momentum $f_L = \Delta L/L_b$. This is a measure of how closely angular momentum is conserved in the collision.

$$f_L = \frac{\Delta L}{L_b} = \frac{0.1 \times 10^{-3}\ kg\text{-}m^2/s}{5.0 \times 10^{-3}\ kg\text{-}m^2/s} = 2.0 \times 10^{-3}$$

. This is pretty amazing considering the difficulty in reading data points for voltage. This result is in fact misleading. Our errors are much greater than this presumed "exactness" of the conservation of L.

i) Based on your calculation in part d), calculate the external frictional torque (due to the motor) during the rotational collision. Do you need to average your results from part d)?

$$\tau_{ave} = (\tau_1 + \tau_2)/2 = 1.0 \times 10^{-3}\ J$$

j) Calculate the torque times the collision time, rotational impulse, $\mathcal{J} = \tau \Delta t_{collision}$. How does this compare to your calculation for the change in angular momentum during the rotational collision in part g)?

$$J_{rotational\ impulse} = \tau \Delta t_{collision} = (1.0 \times 10^{-3}\ J)(1.1\ s) = 1.1 \times 10^{-3}\ J\text{-}s$$

This is 10 times greater than ΔL. There are several reasons for this. The actual collision time is much shorter that 1.1 sec. It is probably 1/2 that time. Then our estimates for when the collision began and ended are contributing to errors in V_{before} and V_{after}. Still we see that angular momentum is nearly conserved.