

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X

Fall Term 2002

Experiment Flow Analysis

Handed out: November 26

Due: Dec 6 at 4 pm in 4-339B

Problem 1: Here is a summary of the measurements, graphs, and calculations you will perform in the lab Experiment Flow.

- a) Record room and water temperatures at the start and finish of your experiment.
- b) Record the length of your stirrers.
- c) On the linear graph paper, plot the level of the water above the hole (the "head") in mm versus the average time in seconds to reach that level for each of the tube lengths. (You should have three graphs.)
- d) On the semi-log paper, plot the head vs. average time for each of the tube lengths.
- e) Determine the time constants for each of your three experiments. Report your results in the table **Time Constants for Experiment Flow** that is attached to the problem.
- f) Make a plot of tube length vs. time constant. Is there any nice curve that passes through the data points? What does an extrapolation to zero tube length mean?
- g) Calculate the viscosity of water at your measured temperature for each of the three pipe lengths.
- h) Briefly summarize your error analysis.

REPORTING THE DATA

Your linear graph of head vs. average time should be an exponentially decaying function,

$$h(t) = h_0 e^{-at}$$

where h_0 is the value of the head at $t = 0$ and a is a constant. The time constant t associated with this exponentially decay is defined to be the time that it takes for the head to reach a value of $h(t) = h_0 e^{-1} = h_0 / 0.368$. Since $h(t) = h_0 e^{-at}$. The time constant t is related to the constant a according to

$$at = 1 \text{ or } t = 1/a .$$

Your semi-log plot should be nearly a straight line. The natural logarithm of $h(t) = h_0 e^{-at}$ is

$$\ln h(t) = \ln h_0 - a t .$$

So a plot of $\ln h(t)$ vs. time t will be a straight line with

$$\text{slope} = -a = -1/\tau .$$

FINDING THE TIME CONSTANT:

Method 1: Obtain the time constant τ for the flows with the three tube lengths by the following procedure that will use results from both your semi-log graph or your linear graph.

- Use your best straight line in the semi-log paper graph to determine the value of the head, h_0 , at $t = 0$ for each of the three experiments. (Note: if you just choose your initial value from your data sheet you are ignoring the rest of your data values.)
- You can obtain the time constant from the linear graph of head vs. average time by directly reading off the time that the head reaches the value $h_0 / 0.368$. Determine the time constants for each of your three experiments.
- **Report your results in the table Time Constants for Experiment Flow that is attached to the write-up.**

Method 2: From your semi-log plot, calculate the slope of your best fit straight line. Compute the time constant according to $\text{slope} = -a = -1/\tau$.

Make a plot of tube length vs. time constant. Is there any nice curve that passes through the data points? What does the extrapolation to zero tube length mean?

Tube Length vs. Time Constant

Make a plot of tube length vs. time constant on linear graph paper. Is there a regular curve that passes through the data points? Consider what does it mean to extrapolation to zero tube length. Briefly summarize your conclusions about your time constants and your plot of tube length vs. time constant.

Viscosity analysis

In Experiment Flow, you made a plot of water level height h vs. time t . Here's how to interpret your data in terms of the coefficient of viscosity. The volume flow rate of water in your bottle is given by

$$\frac{V_t}{t} = -A \frac{dh}{dt} \quad (1)$$

where A is the cross sectional area of your water bottle. According to Poiseuille's Equation, the volume flow rate from the pipe is given by

$$\frac{V_t}{t} = \frac{\rho(P_1 - P_2)R^4}{8\eta L} \quad (2)$$

where L is the length of the pipe, R is the radius of the pipe, η is the coefficient of viscosity, $P_2 \equiv P_{atm}$, is the atmospheric pressure at the open end of the pipe, and P_1 is the pressure at the opening of the pipe inside the bottle. The pressure P_1 is given by Pascal's Law

$$P_1 = P_{atm} + \rho gh \quad (3)$$

where $\rho = 1.0 \times 10^3 \text{ kg/m}^3$ is the density of water, $g = 9.8 \text{ m/s}^2$, and h is the height of the water level above the hole. Thus $P_1 - P_2 = \rho gh$.

- a) Setting eq.'s (1) and (2) equal yields a differential equation for the rate of change of the height of the water as a function of time

$$\frac{dh}{dt} = -\frac{\rho(P_1 - P_2)R^4}{8\eta LA} \quad (4)$$

Using Pascal's Law (3) for the pressure difference in equation (4) then gives

$$\frac{dh}{dt} = -\frac{\rho g h R^4}{8hLA}$$

This is a first order linear differential equation for the height and can be integrated by the techniques of separation of variables:

$$\int_{h_0}^{h(t)} \frac{dh}{h} = -\int_0^t \frac{\rho g R^4}{8hLA} dt$$

Performing the integration of equation, we have

$$\ln \frac{h(t)}{h_0} = -\frac{\rho g R^4}{8hLA} t \tag{5}$$

Now exponentiate both sides of eq. (5) to derive the height as a function of time:

$$h(t) = h_0 \exp\left(-\frac{\rho g R^4}{8hLA} t\right) \tag{6}$$

By making a semi-log plot of the height vs. average time, the coefficient of viscosity can be calculated from the slope of the best-fit straight line according to

$$h = -\frac{\rho g R^4}{8LA(\text{slope})} \tag{7}$$

Calculating the Viscosity of Water:

Using eq. (7), calculate your experimental values for the viscosity of water for each length of pipe **using the data you obtained in Experiment Flow**. What was the temperature of the water for your measurements? The coefficient of viscosity varies as a function of the temperature of the water.

Error Analysis

What are the sources of error in this experiment? Think carefully about your various measurements. Here are some questions to consider: Are all your measurements equally reliable? What was the flow like when you took your last measurement in a particular set? Was it similar to the flow at other times? Is there a progressive change in the times to reach each level as you

make runs? (that might have to do with gas dissolved in the water) How did you make your first measurement? How accurately could you measure the various quantities in the experiments? Were changes in tube diameter significant? You may have noticed something else; tell us.

Data Sheet for FLOW EXPERIMENT

Expt. I: Length of tube =

Room Temperature: Start _____ Finish _____

Water Temperature: Start _____ Finish _____

Time of Day: _____

Water level (mm)	T ₁ (sec)	T ₂ (sec)	T ₃ (sec)	T _{ave} (sec)

Notes:

Data Sheet for FLOW EXPERIMENT

Expt. II: Length of tube =

Room Temperature: Start _____ Finish _____

Water Temperature: Start _____ Finish _____

Time of Day: _____

Water level (mm)	T ₁ (sec)	T ₂ (sec)	T ₃ (sec)	T _{ave} (sec)

Notes:

Data Sheet for FLOW EXPERIMENT

Expt. III: Length of tube =

Room Temperature: Start _____ Finish _____

Water Temperature: Start _____ Finish _____

Time of Day: _____

Water level (mm)	T ₁ (sec)	T ₂ (sec)	T ₃ (sec)	T _{ave} (sec)

Notes:

Time Constants for Experiment Flow

Trial	Tube length (mm)	Time constant (sec)	Container diameter (mm)	Temperature start ($^{\circ}\text{C}$)	Temperature finish ($^{\circ}\text{C}$)
Experiment I					
Experiment II					
Experiment III					

Notes: