Fall 2002

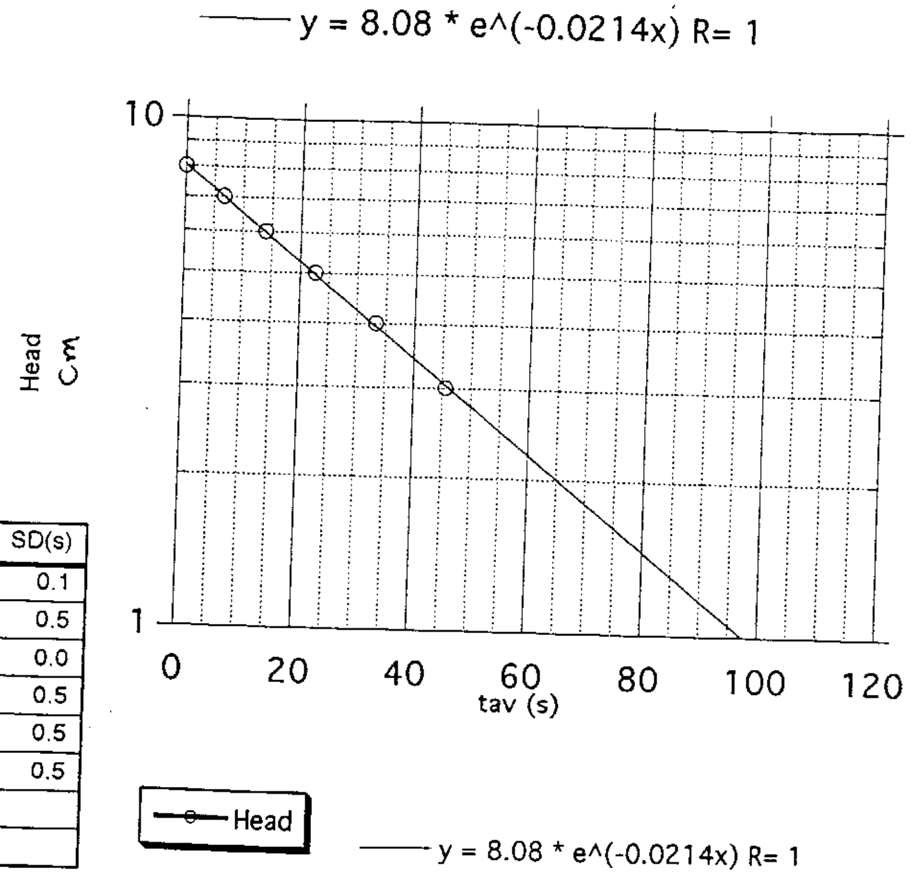
Data and Graphs for L = 131 mm

FL — 8.01X

Expt FLOW DATA

slope $= \dfrac{\ln(40) - \ln(81)}{(32.5 - 0)\,s}$

$= -.0217 \ \frac{1}{s}$



	Head cm	t1 (s)	t2 (s)	t3 (s)	tav (s)	SD(s)
0	8	0.1	0.1	0.1	0.1	0.1
1	7	7	6	7	6.7	0.5
2	6	14	14	14	14.0	0.0
3	5	22	23	23	22.7	0.5
4	4	34	33	33	33.3	0.5
5	3	46	45	46	45.7	0.5
6	1					
7	0					

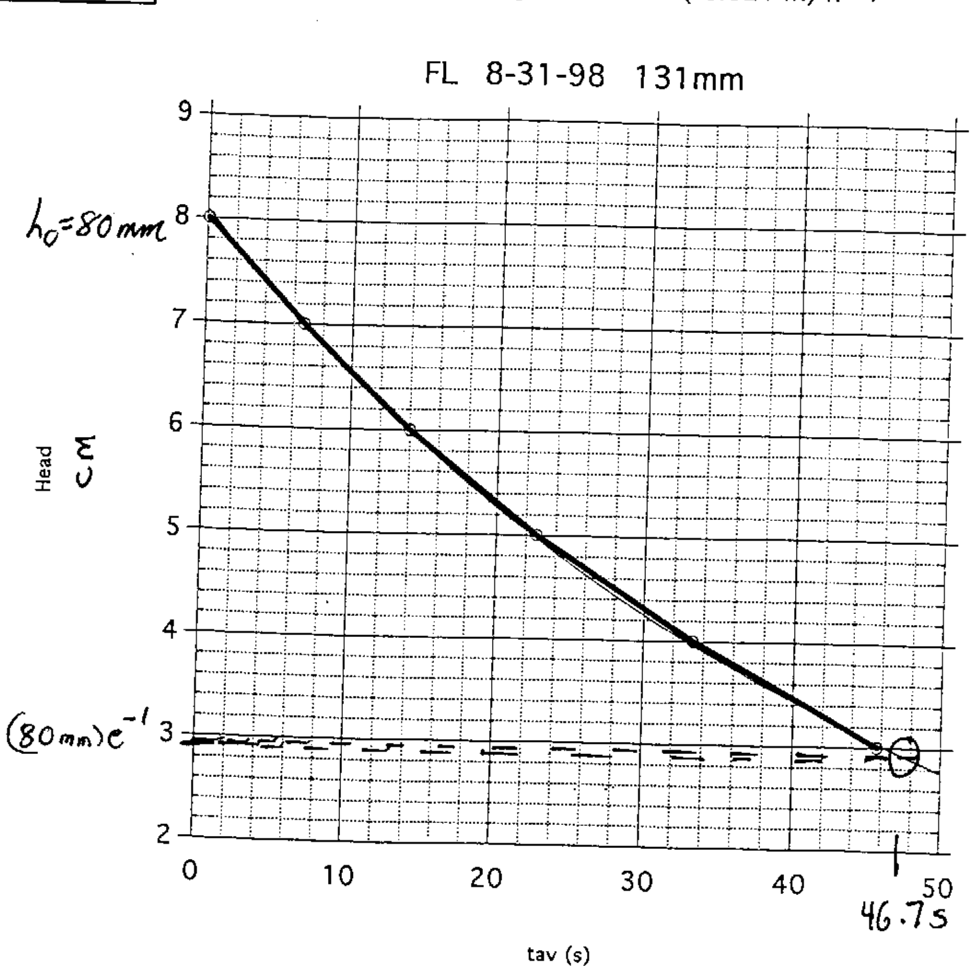
time constant

$\tau = \dfrac{1}{\alpha} = \dfrac{1}{.0214}\ s$

$= 46.7\ s$

$h(t=\tau) = h_0 e^{-1}$

$= 46.7\ s$

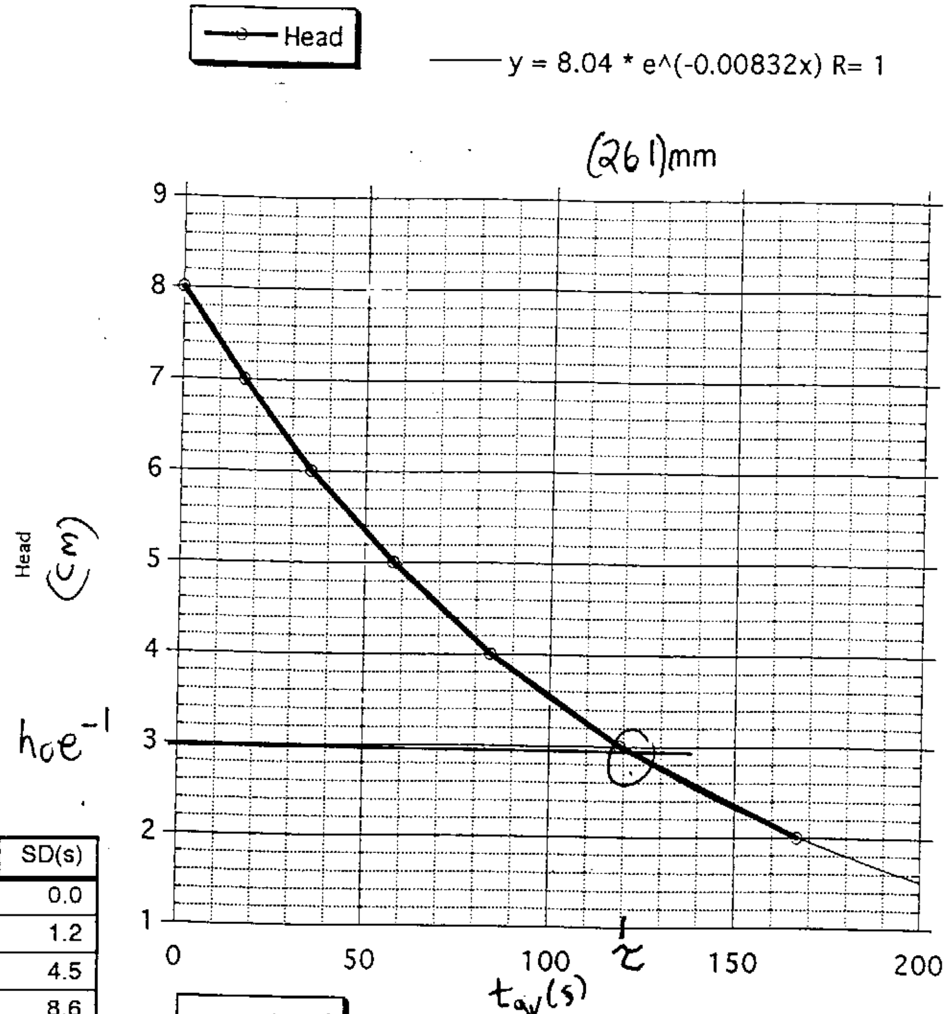


Data Chart + Graphs for

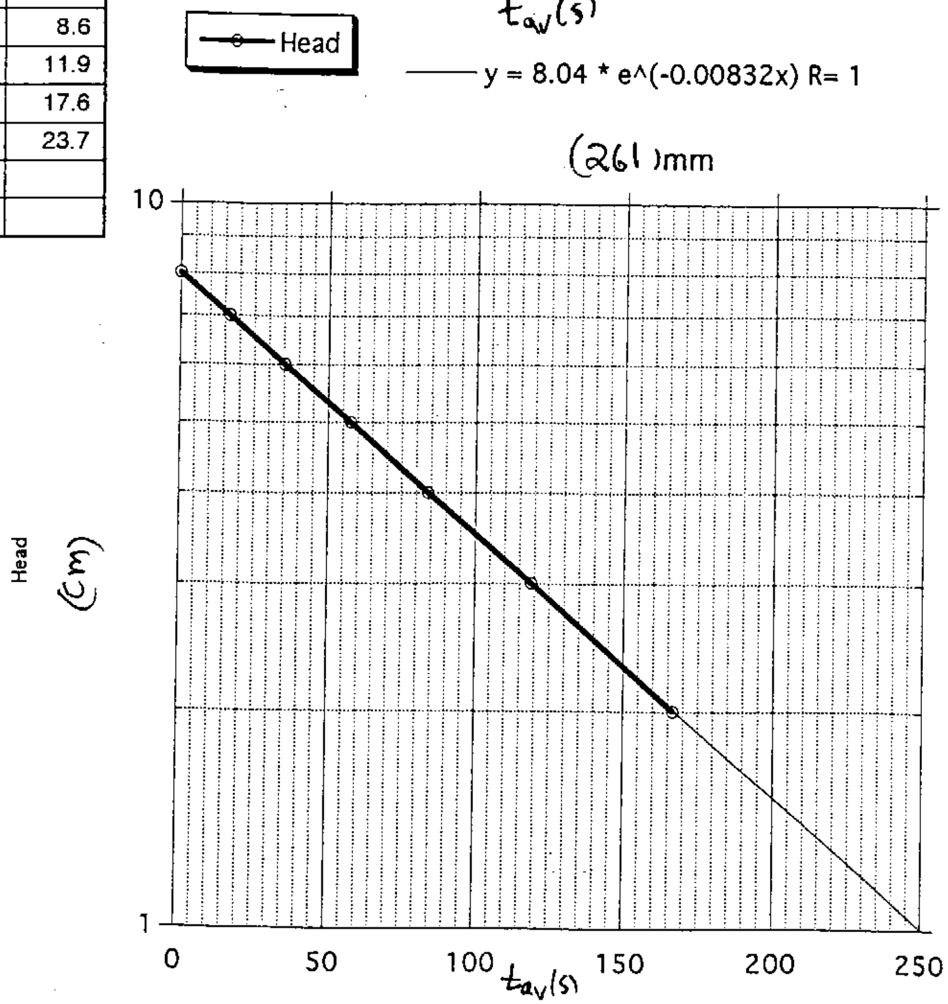
$L = 261$ mm

$$\text{Slope} = \frac{\ln(50) - \ln(80.4)}{(57.5 - 0)\,s}$$

$$= -.00826 \ \frac{1}{s}$$



	Head cm	t1 (s)	t2 (s)	t3 (s)	tav (s)	SD(s)
0	8	0	0	0	0.0	0.0
1	7	15	17	18	16.7	1.2
2	6	30	35	41	35.3	4.5
3	5	47	58	68	57.7	8.6
4	4	70	83	99	84.0	11.9
5	3	97	120	140	119.0	17.6
6	2	138	166	196	166.7	23.7
7	1					
8	0					

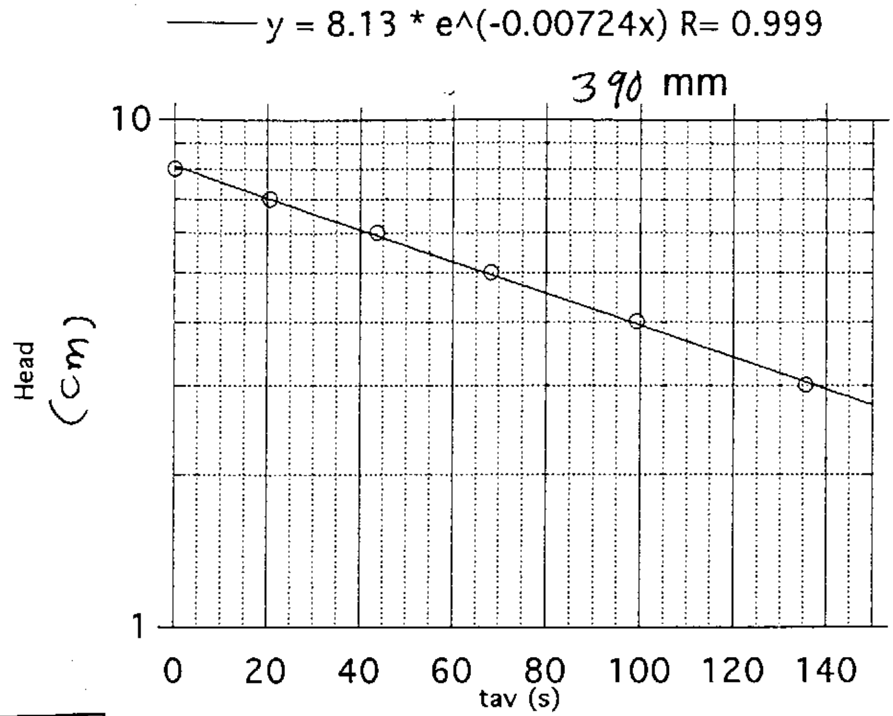


$$\tau = \frac{1}{\alpha} = \frac{1}{0.00832}$$

$$= 120 \ s$$

$$h(\tau) = 8.04\, e^{-1} = 2.96\text{cm}$$

Data and Graphs
for $L = 390$ mm



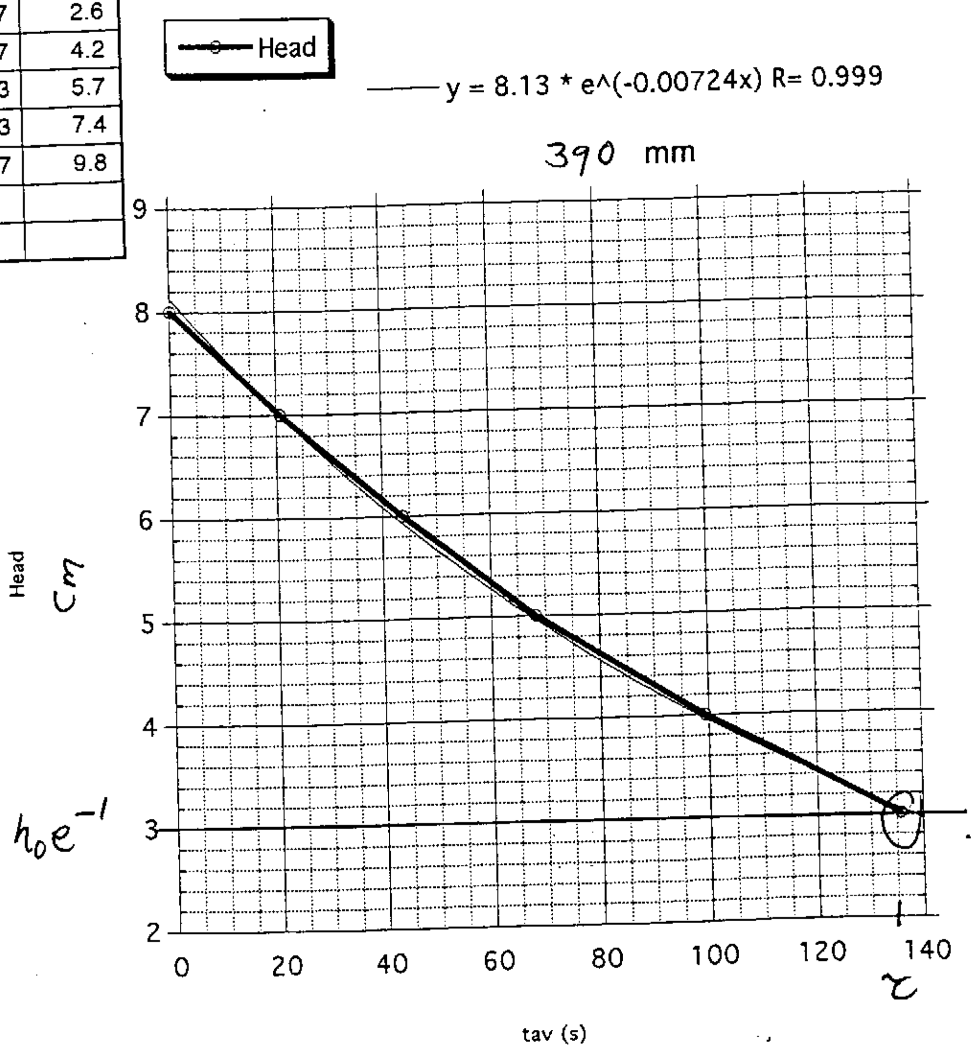
$$\text{slope} = \frac{\ln(70) - \ln(81.0)}{(21 - 0)\,s}$$

$$= -6.95 \times 10^{-3}\ \frac{1}{s}$$

my computer plotted a slope

$$= -7.24 \times 10^{-3}\ \frac{1}{s}$$

	Head cm	t1 (s)	t2 (s)	t3 (s)	tav (s)	SD(s)
0	8	0	0.1	0.1	0.1	0.1
1	7	17	22	23	20.7	2.6
2	6	38	45	48	43.7	4.2
3	5	61	69	75	68.3	5.7
4	4	90	100	108	99.3	7.4
5	3	123	137	147	135.7	9.8
6	1					
7	0					



time constant

$\tau = 138$ s

$h(\tau) = h_0 e^{-1} = (8.13)\, e^{-1}$

$= 30$ mm

Viscosity Calculations:

$R = \frac{2.8}{2} \times 10^{-3}\ m \quad , \rho = 1.0 \times 10^{3} \frac{kg}{m^3}$

$A = \pi\left(\frac{d}{2}\right)^2$ where $d = 63.5 \times 10^{-3}\ m$

$A = 3.17 \times 10^{-3}\ m^2$

$$\eta = \frac{-\pi \rho g R^4}{A\, 8 L\, (\text{slope})} \quad , \text{slope} = -\alpha$$

$$\eta_1 = \frac{\pi \rho g R^4}{A\, 8 L_1\, \alpha_1} = \frac{(\pi)(1.0 \times 10^{+3} \frac{kg}{m^3})(9.8 \frac{m}{s^2})(1.4 \times 10^{-3}\ m)^4}{(3.17 \times 10^{-3}\ m^2)(8)(0.131\ m)(0.0214 / s)}$$

$= 1.7 \times 10^{-3}$ Pa-s

$$\eta_2 = \frac{(\pi)(1.0 \times 10^{+3} \frac{kg}{m^3})(9.8 \frac{m}{s^2})(1.4 \times 10^{-3}\ m)^4}{(3.17 \times 10^{-3}\ m^2)(8)(0.261\ m)(.00832 / s)} = 2.1 \times 10^{-3}\ Pa\text{-}s$$

$$\eta_3 = \frac{(\pi)(1.0 \times 10^{3} \frac{kg}{m^3})(9.8 \frac{m}{s^2})(1.4 \times 10^{-3}\ m)^4}{(3.17 \times 10^{-3}\ m^2)(8)(0.390\ m)(.00724\ s^{-1})} = 1.7 \times 10^{-3}\ Pa\text{-}s$$

Since the viscosity

$$\eta = \frac{-\pi \rho g R^4}{A 8 L} \frac{1}{(\text{slope})}$$

$$\frac{1}{\text{slope}} = \frac{1}{(-\alpha)} = -\tau$$

τ = Time constant

$$\eta = \frac{\pi \rho g R^4 \tau}{A 8 L}$$

$$\tau = \frac{A 8 \eta L}{\pi \rho g R^4}$$

$$\text{slope} = \frac{A 8 \eta_{ave}}{\pi \rho g R^4}$$

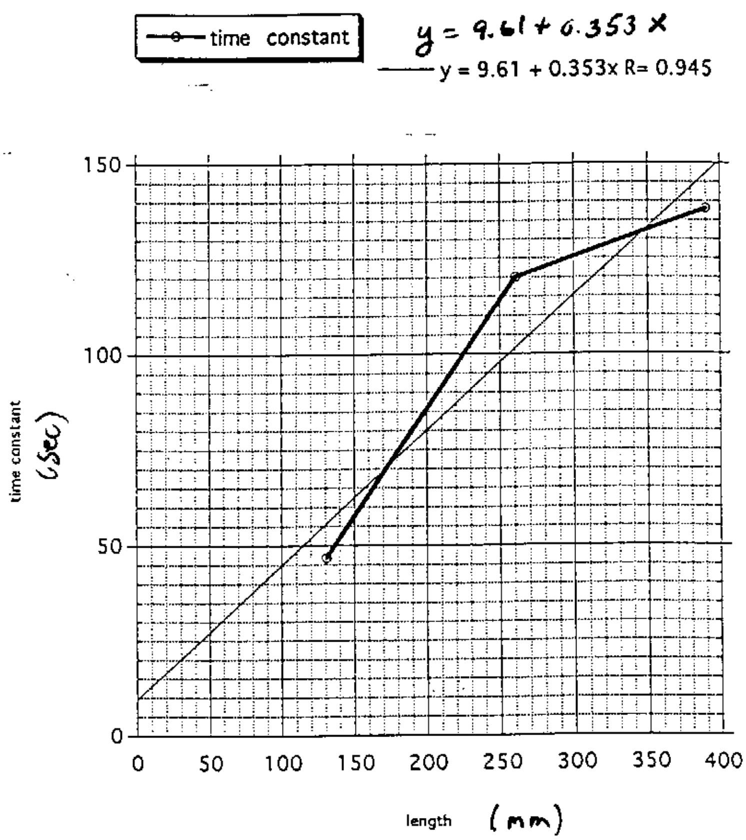
$$\eta_{ave} = \frac{\pi \rho g R^4 (\text{slope})}{A 8}$$

$$= \frac{(.353 \times 10^{-3} \tfrac{s}{m})(\pi)(1.0 \times 10^3 \tfrac{kg}{m^3})(9.8 \tfrac{m}{s^2})(1.4 \times 10^{-3} m)^4}{(3.17 \times 10^{-3} m^2)(8)}$$

$$\eta_{ave} = 1.6 \times 10^{-3} \text{ Pa-s}$$

	length	time constant
0	131	46.7
1	261	120
2	390	138

$y = 9.61 + 0.353x$



notice that when $L=0$ $\Rightarrow \tau = 0$ but this is unphysical since $L=0$ is the case of a hole and it takes a finite amount of time to empty the bottle.
Laminar flow is not established for short L
Turbulence dominates!

Thus $\eta = \dfrac{-\pi \rho g R^4}{A\,8\;L(\text{slope})}$. I measured the radius of the pipe to be $R = 1.5 \times 10^{-3}$ m. This is not very accurate. and is the greatest source of error for my value of η because

$\dfrac{\Delta\eta}{\eta} \cong 4\dfrac{\Delta R}{R}$ since R is raised to the fourth power. If $\Delta R \sim .2 \times 10^{-3}$ m, $\dfrac{\Delta R}{R} \simeq \dfrac{.2 \times 10^{-3}\,m}{1.5 \times 10^{-3}\,m} \simeq .13$

and $\dfrac{\Delta\eta}{\eta} \simeq = .53$. 50% error.

The measured diameter for the water bottle $d = 63.5 \times 10^{-3}$ m

So $A = \pi\left(\dfrac{d}{2}\right)^2$. Thus if $\Delta d \simeq 1.0 \times 10^{-3}$ m

$\dfrac{\Delta A}{A} \simeq 2\dfrac{\Delta d}{d}$ and since we need A^2

$\dfrac{\Delta A}{A} \simeq 2\dfrac{\Delta d}{d} \simeq (2)\left(\dfrac{1.0 \times 10^{-3}\,m}{63.5 \times 10^{-3}\,m}\right) \simeq .031$ or 3.1%. A much smaller error

$\dfrac{\Delta L}{L} \simeq \dfrac{.5\,mm}{129\,mm} \simeq 3.9 \times 10^{-3}$ or .39% (negligible)

so $\left(\dfrac{\Delta\eta}{\eta}\right|_{total} \simeq \left(\left(\dfrac{2\Delta d}{d}\right)^2 + 4\left(\dfrac{\Delta R}{R}\right)^2\right)^{1/2} \simeq .53$

So the major source of measurement error is due to errors in the value of the pipe diameter