MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.01X — Fall Term 2002

## Solutions to Problem Set #1

**Problem 1:** The Hydrogen Maser (Y&F 1-53)

Note: $1,420,405,751.786 \approx 1.42 \times 10^9$, to 3 significant figures.

a) The time for one cycle is $(1.42 \times 10^9)^{-1} = 7.04 \times 10^{-10}$ s.

b) The number of cycles in 1 h is $(1.42 \times 10^9) \times 3600 = 5.11 \times 10^{12}$.

c) During the age of the earth there would have been $(1.42 \times 10^9) \times (3.16 \times 10^7) \times (4.6 \times 10^9) = 2.1 \times 10^{26}$ cycles. (You should only give 2 significant figures, as that is all we have for the age of the earth. It may be useful to remember that a year is approximately $\pi \times 10^7$ s.)

d) If it is off by 1 s in $10^5$ years, it would be off by $1 \times (4.6 \times 10^9)/(10^5) = 4.6 \times 10^4$ s over the age of the earth. That is about 13 hours.

**Problem 2:** Fermi Estimations (Y&F 1-20, 1-26)

1-20) Suppose you breathe 12 times a minute, on average; this works out to 360 liters/hr, or $3.1 \times 10^3\,\text{m}^3/\text{year}$, hence $2.2 \times 10^5$ m$^3$ in your allotted three-score and ten. Unlike the Astrodome, Fenway park lacks a roof so its volume is less well defined. The field is roughly a square 100 m on a side. Let's make that 200 m on a side when the stands are included and assume an average height of 50 m. The volume is then about 2 million m$^3$, roughly 10 times what you would breathe in 70 years. (It is important when making estimates like this to state your assumptions. If you make different assumptions, you will get a different answer.)

1-26) You need to decide how thick a crisp new dollar bill is; 0.005 inch, or 125 $\mu$m, seems reasonable. The distance to the moon is 240,000 mi or about $3.9 \times 10^8$ m. This would be the thickness of $3 \times 10^{12}$ dollar bills. The Apollo program cost about 50 billion dollars, significantly cheaper, and required killing many fewer trees than printing $3 \times 10^{12}$ dollar bills would.

Here's a Fermi question for you: is my statement about killing fewer trees a reasonable one? (You might want to estimate how much paper was used for for Apollo program documents; remember that the program predated PCs and laser printers.)

**Problem 3:** Relay Race (Y&F 2-48)

a) Edith runs the first 25.0 m in 20.0 s, so her average speed is $25.0/20.0 = 1.25$ m/s.

b) She runs the return trip of 25.0 m in 15.0 s, so her average speed is $25.0/15.0 = 1.67$ m/s.

c) She ends up where she started, so her displacement vector is zero. Thus, no matter what speed she runs the two paths at, her average velocity vector is zero.

d) She covers 50.0 m in a total of 35.0 s, so her average speed for the round trip is $50.0/35.0 = 1.43$ m/s.

**Problem 3:** (continued)
Note: you will not get the correct answer to (d) by taking the simple arithmetic mean, $(1.25+1.67)/2 = 1.46$, of your answers to (a) and (b) because Edith ran at these two speeds for different amounts of time. What you would have to calculate is an average where each speed is weighted by the fraction of the total time Edith ran at that speed. Thus

$$S_{av} = \left(\tfrac{20.0}{35.0}\right) 1.25 + \left(\tfrac{15.0}{35.0}\right) 1.67$$

does give the correct answer. It is much easier to simply divide the total distance by the total time to get the average speed.

**Problem 4:** Passing a Police Car (Y&F 2-60)

a) Over the time from when the truck first passes the police car and when the police car passes the decelerating truck, both vehicles travel the same distance. Therefore they both have the same average speed. Thus the truck's average speed must be the same as that of the police car, $v_p$. The initial speed was $\frac{3}{2}\,v_p$ and the constant deceleration means the average speed is just the arithmetic mean of the initial and final speeds, so the final speed must be $\frac{1}{2}\,v_p$. This does not depend on the magnitude of the deceleration, only on the fact that it is constant.

b) Suppose the truck passes the police car at time $t_1$ and at position $x_1$. The position of the police car is $x_p = x_1 + v_p(t - t_1)$ for all times. The position of the truck is given by $x_t = x_1 + \frac{3}{2}v_p(t - t_1)$ for $t \leq t_1$ and by $x_t = x_1 + \frac{3}{2}v_p(t - t_1) - \frac{1}{2}a(t - t_1)^2$ for $t_1 \leq t \leq t_3$, where $t_3$ is the time the truck stops and $a$ is the magnitude of the truck's deceleration. The velocity of the truck is $v_t = \frac{3}{2}v_p - a(t - t_1)$ for $t \geq t_1$. From this you can see that the truck will stop at a time $t_3 = t_1 + \frac{3}{2}v_p/a$; for $t \geq t_3$ the value of $x_t$ will be constant.

You could use these equations to answer part (a), but the method used above is easier. From the equations you can find the police car passes the truck at $t_2 = t_1 + v_p/a$; thus the truck is still moving when it is passed by the police car. If you were to graph $x$ as a function of $t$, you would get something like (dashed line is $x_t$, solid is $x_p$):