

Solutions for 8.01x Problem Set 10

8-79: Following the suggestions in the book, one can first show that the angle between the two outgoing pucks is 90° . Momentum conservation yields two equations:

$$mv_{A,i} = mv_{A,f} \cos \alpha + mv_{B,f} \cos \beta \quad (1)$$

$$0 = mv_{A,f} \sin \alpha + mv_{B,f} \sin \beta \quad (2)$$

If we square and add the two first equations and eliminate the mass, we obtain

$$v_{A,i}^2 = v_{A,f}^2 + v_{B,f}^2 + 2v_{A,f}v_{B,f}(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \quad (3)$$

$$= v_{A,f}^2 + v_{B,f}^2 + 2v_{A,f}v_{B,f} \cos(\alpha + \beta) \quad (4)$$

$$(5)$$

Energy conservation gives

$$v_{A,i}^2 = v_{A,f}^2 + v_{B,f}^2 \quad (6)$$

Subtracting the two equations we obtain

$$0 = 2v_{A,f}v_{B,f} \cos(\alpha + \beta) \text{ and therefore} \quad (7)$$

$$\alpha + \beta = \pi/2. \quad (8)$$

Knowing this, we see that the angle for puck B is 65° . Momentum conservation in y direction gives $v_{B,f} = 0.466v_{A,f}$. Momentum conservation in x gives the final speed of puck A as $v_{A,f} = v_{A,i}/(\cos(25^\circ) + 0.466 \cos(65^\circ)) = 13.6 \text{ m/s}$. For puck B we get $v_{B,f} = 6.34 \text{ m/s}$.

9-6: By differentiating $\Theta(t)$, we obtain

$$\omega(t) = (250 \text{ rad/s}) - (40.0 \text{ rad/s}^2)t - (4.50 \text{ rad/s}^3)t^2 \text{ and}$$

$$\alpha(t) = (40.0 \text{ rad/s}^2) - (9.00 \text{ rad/s}^3)t.$$

for the angular velocity ω and the angular acceleration α .

a) $\omega(t) = 0$ for $t = 4.23\text{s}$.

b) At $t = 4.23$, $\alpha(t) = -78.1 \text{ rad/s}^2$.

- c) At $t = 4.23$, $\omega(t) = 586$ rad, corresponding to $586/(2\pi) = 93.3$ revolutions.
 d) At $t = 0$, $\omega(t) = 250$ rad/s.
 e) The average ω is $\omega_{ave} = 586 \text{ rad} / 4.23\text{s} = 138 \text{ rad/s}$.

9-36: For the slender rod, the moment of inertia is $I = \frac{1}{12}mL^2$. The kinetic energy is therefore

$$K_{rot} = \frac{1}{2} \frac{1}{12} mL^2 \omega^2 = 1.3 \times 10^6 \text{ J}.$$

To gain the same kinetic energy in a free fall (neglecting air resistance), the object needs to drop a height

$$h = K/(mg) = 1160 \text{ m}.$$

16-25:

- a) Average kinetic energy $K_{ave} = 3/2kT = 6.21 \times 10^{-21} \text{ J}$.
 b) Average speed squared: $v_{ave}^2 = 2K_{ave}/m = 2.34 \times 10^5 \text{ m}^2/\text{s}^2$.
 c) RMS speed : $v_{rms} = \sqrt{\frac{3RT}{M}} = 4.84 \times 10^2 \text{ m/s}$.
 d) Momentum : $p = m v_{rms} = 2.57 \times 10^{-23} \text{ kg m/s}$.
 e) Average force is obtained from the change in momentum of the molecules per collision, divided by the time between collisions, as $J = F_{ave} \cdot \Delta t$. Average momentum change is $2 \times p$, average time is $2 \times L/v$, where L is the length of the container. This gives

$$F_{ave} = \frac{2mv_{rms}}{2L/v_{rms}} = mv_{rms}^2/L = 1.24 \times 10^{-19} \text{ N}.$$

- f) Pressure per molecule: $P_{ave} = F_{ave}/L^2 = 1.24 \times 10^{-17} \text{ Pa}$.
 g) $N = P/P_{ave} = 8.15 \times 10^{21}$ molecules.
 h) Number of molecules $N = nN_A = N_A PV/(RT) = 2.45 \times 10^{22}$.
 i) The discrepancy arises from assuming in g) that all molecules move in the same direction (one dimension), whereas in reality their motion is three-dimensional.