Massachusetts Institute of Technology
Physics Department

Physics 8.01x — Fall term 2002

Solutions to Problem Set# 11

Problem 1) Y&F 9-66, p291

The equation for moment of inertial is an integral over the volume, and if the axis is along z-axis, it is

$$I = \int_{vol} \rho(x, y, z)(x^2 + y^2)dV \quad (dV = dxdydz)$$

If we are comparing I of objects of the same mass but different spatial distribution of the mass, the problem becomes, what is the "average" distance of the material to the axis(if $\hat{z}$ is the axis, the distance is $\sqrt{x^2 + y^2}$ of course). This can be done by comparing the cross section shown in figure 9-23.

a) The object with the smallest I must be "compact" so that the material is relatively close to the ration axis. Obviously B's cross section is the least "compact"; out of A and C, because we can put a circle of radius R inside a square of dimention 2R×2R, A is more "compact" than C. So A has the smallest I.

b) As stated in a) B is the least "compact", and hence has the largest moment of inertia. $I_A < I_C < I_B$

c) We can put the solid sphere into A, and A into C. Because B is hollow, it is still the least "compact". Therefore the "compactness" is in the order $sphere > A > C > B$, and the order of moment of inertial is $sphere < A < C < B$.

Problem 2) Y&F 9-82, p293

a) The rotational kinetic energy with period T and moment of inertia I, and rate of energy loss is

$$\begin{aligned} \omega &= \frac{2\pi}{T} \Longrightarrow KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{2\pi}{T}\right)^2 \\ \frac{d}{dt}KE_r &= \frac{d}{dt}\left[\frac{1}{2}I\left(\frac{2\pi}{T}\right)^2\right] = -I\frac{(2\pi)^2}{T^3}\frac{dT}{dt} \end{aligned}$$

From the conditions in the problem $dT/dt = 4.22 \times 10^{-13} s/s$ we have

$$\frac{d}{dt}KE_r = -5 \times 10^{31}W = -I\frac{(2\pi)^2}{T^3}\frac{dT}{dt} \Longrightarrow I = 1.09 \times 10^{38} kg \cdot m^2.$$

b) Moment of inertial for a solid uniform sphere is $I = \frac{2}{5}Mr^2$, where M is total mass and r the radius. From a) we have

$$r = \left(\frac{5}{2}I/M\right)^{1/2} = 9882m.$$

c) We already know the angular frequency of the neutron star $\omega = 2\pi/T$, so the linear speed is, with r from b)

$$v = \omega r = 1.88 \times 10^6 m/s.$$

d)

$$\begin{aligned} \rho &= M/\left(4\pi/3r^3\right) = 1.4M_{sun}/\left(4\pi/3r^3\right) = 6.89 \times 10^{17} kg/m^3 \\ \frac{\rho}{\rho_{rock}} &\approx 10^{14}, \qquad \frac{\rho}{\rho_{nucl}} \approx 7 \end{aligned}$$

so the density of the neutron star is orders greater than that of rock, but of the same order that of an atomic nucleus. This means a neutron star is essentially a large atomic nucleus.

Problem 3) Y&F 10-30, p322

a) A point-like mass in circular motion has angular momentum

$$L = |r \times p| = mvr = 2.67 \times 10^{40} kg \cdot m^2/s.$$

where r is the orbital radius and v the orbital speed.

b) The angular momentum of a uniform solid sphere due to self-rotation, in analogy to $p = mv$, is

$$L = I\omega = \frac{2}{5} m r_E^2 \frac{2\pi}{T} = 7.07 \times 10^{33} kg \cdot m^2/s$$

where r_E is the radius of the earth, and the period T is one day.

Problem 4) Y&F 10-33, p322

a) Angular momentum will change when and only when there is net torque applied to the object. In this problem the table's normal force and gravity cancel out; the string's tension force goes through the center of the circle ($\vec{r}//\vec{T}$) so its torque $\vec{\tau} = \vec{r} \times \vec{T} = 0$. Angular momentum is therefore conserved.

b) Initial equals final angular momentum:

$$L_i = m\omega_i r^2 = L_f = m\omega_f (r - \Delta r)^2 \Longrightarrow \omega_f = \omega_i \left(\frac{r}{r - \Delta r} \right)^2 = 7 rad/s.$$

c)

$$\Delta KE = \frac{1}{2} m (r - \Delta r)^2 \omega_f^2 - \frac{1}{2} m r^2 \omega_i^2 = 0.0103 J.$$

d) By the work-kinetic-energy theorem the work done by the cord is

$$W = \Delta KE = 0.0103 J.$$