Solutions for 8.01x Problem Set 2

2-68: Let h be the height of the window, Δt time the flowerpot takes to pass the window and d the distance between the windowsill where the flowerpot started and the top of the window below. (using a coordinate system where x points downwards and the flowerpot starts falling at $t = 0$ and $x = 0$). Then the velocity v_1 of the flower pot when it reaches the top of the window will be related to its average velocity while passing the window like this:

$$v_{av} = v_1 + \frac{1}{2} g \Delta t$$

Using $h = v_{av} \Delta t$, we find

$$\begin{aligned} h &= v_1 \Delta t + \frac{1}{2} g \Delta t^2 \text{ or} \\ v_1 &= h/\Delta t - \frac{1}{2} g \Delta t. \end{aligned}$$

Using $x(t) = \frac{1}{2} g t^2$ and $v(t) = gt$, we find that

$$x(t) = \frac{v(t)^2}{2g}$$

This gives the following expression for d:

$$\begin{aligned} d &= \frac{v_1^2}{2g} = \frac{(h/\Delta t - \frac{1}{2} g \Delta t)^2}{2g} \\ &= 0.310\text{m} \end{aligned}$$

2-72: a. Using a coordinate system where x points upward, $x = 0$ is at ground level and $t = 0$ when the students walks of the top of the building ($h = 180$ m), the student reaches the ground at a time

$$t_{student} = \sqrt{2h/g} = 6.06 \text{ s}.$$

As superman arrives at $t0 = 5$ s, this leaves him 1.06 s to reach the student. Supermans equation of motion is

$$\begin{aligned} x(t) &= h + v_0(t - t_0) - 1/2g(t - t0)^2 \text{ or} \\ v_0 &= (x(t) - h)/(t - t_0) - 1/2g(t - t_0)^2) \end{aligned}$$

To find the velocity v_0 superman needs to reach the ground at the same time as the student, we use $t = 6.06$ s and $x(t) = 0$, which gives $v_0 = -164.3$ m/s. Superman should therefore start with a downward velocity bigger than 164.3 m/s.

b. The graph of the students and supermans trajectory is shown in Fig. 1.

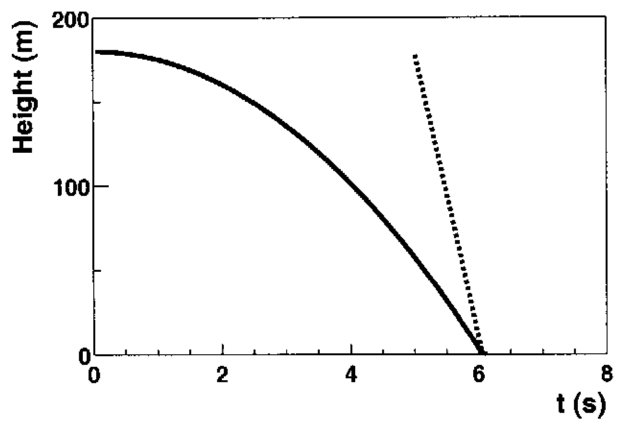

Figure 1: Problem 2-72: $x(t)$ vs t graph for student (solid line) and superman (dashed line).

c. The height of the building has to be such that $t = \sqrt{2h/g} > 5$ s, i.e. $h > 122.5$ m.

3-6: a. The components of the dogs acceleration are

$$\begin{aligned} a_x &= 0.45 \text{ m/s}^2 \cos(31.0°) = 0.39 \text{m/s}^2 \text{ and} \\ a_y &= 0.45 \text{ m/s}^2 \sin(31.0°) = 0.23 \text{m/s}^2 \end{aligned}$$

The resulting velocity components after $\Delta t = t2 - t1$ are therefore:

$$\begin{aligned} v_x &= 2.6 \text{m/s} + 0.39 \text{m/s}^2 \times 10 \text{ s} = 6.5 \text{ m/s and} \\ v_y &= -1.8 \text{m/s} + 0.23 \text{m/s}^2 \times 10 \text{ s} = 0.52 \text{ m/s} \end{aligned}$$

b. $v = \sqrt{(6.5 \text{ m/s})^2 + (0.52 \text{ m/2s})^2} = 6.48$ m/s, at an angle of $\arctan(6.5/0.52) = 85°$.

c. See Fig. 2. The vectors differ by $\Delta\vec{v} = \vec{a}_{av}\Delta t$.

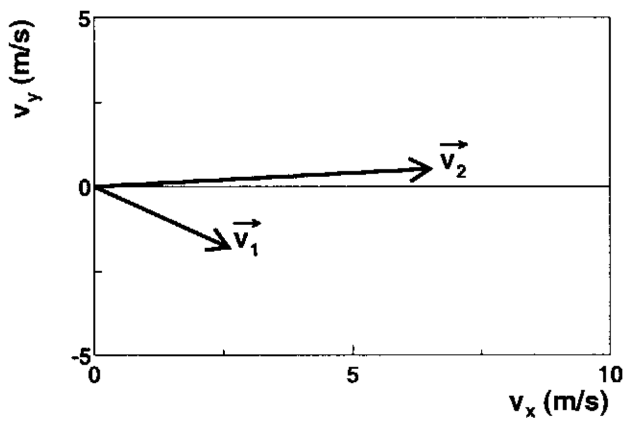

Figure 2: Problem 3-6: Velocity vector for $t = t_1$ and $t = t_2$.

4-10: Starting from $x(t) = \frac{1}{2}at^2$, we see that the acceleration of the block is $a = 2x/t^2 = 0.88$ m/s^2.

a. Using Newton's second law, the mass follows as $m = F/a = 90.9$kg.

b. If no further force is applied, the block will continue to move with constant velocity. The velocity reached during the first 5.00 s is $v = at =$ 4.4 m/s. The resulting displacement in the following 5 s is $\Delta x = vt = 22.0$ m.

5-35: The sum of the forces $\vec{F_1}$ and $\vec{F_2}$ has a y-component of $F_{tot_y} =$ 140 N sin 30° − 100 N sin 60° = −16.6 N, using a right-handed coordinate system. To move the cart into the $+x$ direction, the total force in y-direction needs to be zero. The child therefore has to push with a force $F_y = +16.6$ N.

b. If the child pushes with the minimal necessary force, which has only a component in the y-direction, the force accelerating the cart is given by $F_{tot_x} = 140\ \text{N}\cos 30° + 100\ \text{N}\cos 60°$. Using $m = F_{tot_x}/a_x$, the resulting mass is 85.6 kg. The corresponding weight is $w = 85.6\ \text{kg} * 9.80\ \text{m/s}^2 =$ 840 N.