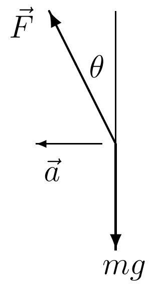
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.01X FallTerm 2002

Solutions to Problem Set #5

Problem 1: Giant Swing (Y&F 5-46)

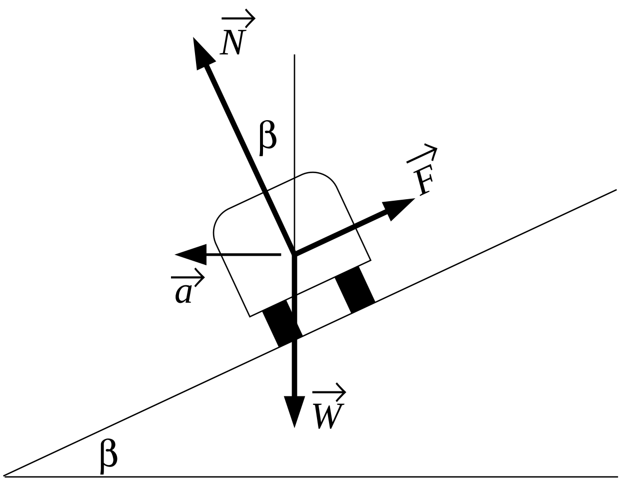
(a) The radius of the circular motion of each seat on the swing is $r = 3.00 + 5.00 \sin 30° = 5.50\,m$, and the centripetal acceleration is $r\omega^2$. A free body diagram of the passenger in a seat is:



The vertical component of $\vec{F}$, $F\cos\theta$, supports the weight of the passenger, so $F\cos\theta = mg$. The horizontal component, $F\sin\theta$, produces the centripetal acceleration $a = r\omega^2$. Eliminating F gives the result $\omega^2 = (g/r)\tan\theta$ so that $T = 2\pi/\omega = 2\pi\sqrt{r/g\tan 30°} = 6.20\,s$.

(b) This is obviously independent of the weight of the passenger.

Problem 2: Rounding a Curve (Y&F 5-91)



A free body diagram of the car is shown at the left. When the car is travelling at speed v around a curve of radius R it has a centripetal acceleration $a = v^2/R$.

The magnitude and direction of the friction force $\vec{F}$ depend on the car's speed. Applying Newton's 2[nd] law to the horizontal and vertical directions we have:

$$mv^2/R = N\sin\beta - F\cos\beta, \text{ and} \qquad (1)$$
$$N\cos\beta + F\sin\beta = W = mg. \qquad (2)$$

The car's speed matches the bank (i.e., $F = 0$) at the speed v_0 where $mv_0^2/R = mg\tan\beta$ or $v_0 = \sqrt{gR\tan\beta}$.

The tires *roll* on the road, so there is no sliding and the appropriate friction coefficient is μ_S. For $v < v_0$, $\vec{F}$ will point up the slope as shown ($F > 0$), and the minimum v will occur when $F = \mu_S N$. When you substitute this into equations (1) and (2) above and eliminate N, you get the result ($\beta = 25°$, $\mu_S = 0.30$, $g = 9.8$):

$v^2_{min} = gR(\tan\beta - \mu_S)/(1 + \mu_S\tan\beta)$ or $v_{min} = 8.5\,m/s$ ($v_{min} = 0$ if $\tan\beta \le \mu_S$).

For $v > v_0$, $\vec{F}$ acts down the slope ($F < 0$) and $F = -\mu_S N$. (However, this is only possible for $\tan\beta < 1/\mu_s$. If $\tan\beta > 1/\mu_S$, $\vec{F}$ will always act up the slope and the car will never slide up the slope for any speed.) Solving gives:

$v^2_{max} = gR(\tan\beta + \mu_S)/(1 - \mu_S\tan\beta)$ or $v_{max} = 21\,m/s$ ($v_{max} = \infty$ if $\tan\beta \ge 1/\mu_S$).

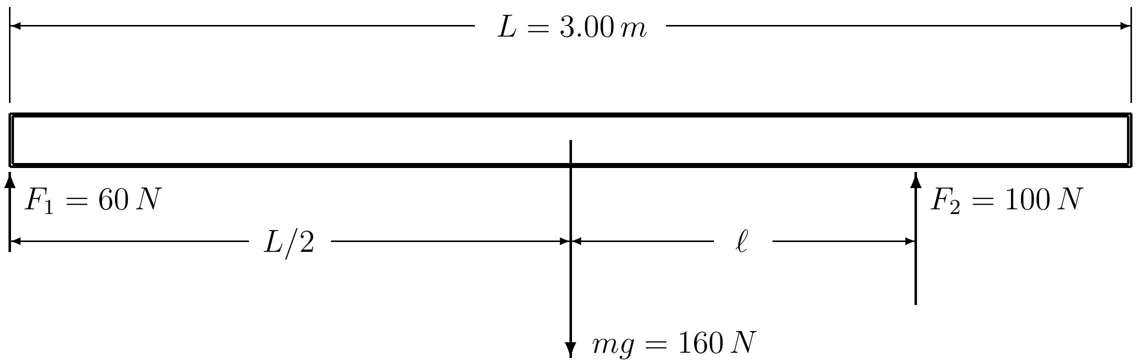
Problem 3: Model Car (Y&F 5-106)

The normal force $\vec{N}$ is always directed towards the center of the circle, as is the acceleration $a = v^2/R$. At the top of the loop the weight of the car is also centripetal, so the centripetal acceleration is caused by $N + mg$. At the bottom of the loop the weight is directed away from the center (centrifugal) and the centripetal acceleration is caused by $N - mg$. Thus:

a) $N = m(v^2/R + g) = 1.60 \times (144/5.00 + 9.80) = 61.8$ newtons (bottom).

b) $N = m(v^2/R - g) = 1.60 \times (144/5.00 - 9.80) = 30.4$ newtons (top).

Problem 4: Carrying a Board (Y&F 11-6)



First of all, we need to draw a free body diagram of the board. The board is a rigid body with a finite size, not a point particle as we have been assuming in the previous problems. For it to be in static equilibrium it is not sufficient for the net vertical and horizontal forces to be zero. The board also must not rotate, which means that the net torque on it must be zero.

We may calculate the torque about any axis we choose. The torque due to a force is the magnitude of the force multiplied by the perpendicular distance from the axis to the line of action of the force. (We began our discussion of vectors several weeks ago with the idea that all vectors with the same magnitude and direction were the same vector; when we consider rigid bodies and torques, that is no longer true. It also matters where the force vector acts.)

Because the board is uniform, we may pretend its weight vector acts at the the geometrical center of the board. The torque calculation is simple if we choose the axis to be perpendicular to the plane of the diagram and pass through the geometrical center of the board. Then the torque due to the weight is zero, that due to F_1 is $-LF_1/2$ (by convention CCW torque is positive) and the torque due to F_2 is ℓF_2. These must cancel, giving $\ell = LF_1/(2F_2)$. The net vertical force will be zero when $F_1 + F_2 = mg$, so we have $\ell = LF_1/(2mg - 2F_1) = 0.90\,m$.

The second person must lift the board with a force of $100\,N$ $2.40\,m$ from the end the first one is lifting or $0.6\,m$ from the other end.