Solutions for 8.01x Problem Set 6

11-17: In this problem, we are again dealing with a system in static equilibrium. There are three unknowns, the tension T_L in the left rope, the tension T_R in the right rope and the angle β between the right rope and the bar. To find these unkowns, we require that $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$. For the torque, we pick the right end of the bar as the pivot point, such that two of the unknowns (T_R and β) don't appear in the first equation. We count counter-clockwise torques as positive. For the overall torque to vanish, we require

$$3.0\text{m} \cdot T_L \cdot \sin 150° - 240\text{N} \cdot 1.5\text{m} - 90\text{N} \cdot 0.5\text{m} = 0$$

which yields $\underline{T_L = 270\text{ N}}$. For the x and y components of the total force we get

$$\begin{aligned} T_L \cdot \sin 150° - 240\text{N} - 90\text{N} + T_{R_y} &= 0 \\ T_L \cdot \cos 150° + T_{R_x} &= 0 \end{aligned}$$

This gives $T_{R_y} = 195$ N and $T_{R_x} = 203.8$ N. The total tension in the right rope is therefore $\underline{T_R = \sqrt{T_{R_y}^2 + T_{R_x}^2} = 304\text{ N}}$, with the angle $\underline{\beta = \arcsin(T_{R_y}/T_R) = 39.9°}$.

Problem 2 - Lifting a weight: Yet another static equilibrium problem. Again, we use $\sum \vec{F} = 0$ and $\sum \vec{\tau} = 0$ to obtain a set of equations. We count counter-clockwise torques as positive and pick the point where F_{disk} acts as the pivot point.

a. That allows us to determine F_{musc}:

$$\frac{2}{3}L \cdot F_{musc} \sin 12° - \frac{1}{2}L \cdot 24\text{kg} \cdot 9.8\tfrac{m}{s^2} \cos 35° - L \cdot 12\text{kg} \cdot 9.8\tfrac{m}{s^2} \cos 35° = 0$$

The required force is $\underline{F_{musc} = 1391\text{ N}}$.

b. F_{disk} can be obtained by requiring that the sum of all forces on the spine vanishes (as it is not accelerating):

$$\begin{aligned} F_{disk_x} - F_{musc} \cos(35° - 12°) &= 0 \\ F_{disk_y} - F_{musc} \sin(35° - 12°) - 24\text{kg} \cdot 9.8\tfrac{m}{s^2} - 12\text{kg} \cdot 9.8\tfrac{m}{s^2} &= 0 \end{aligned}$$

This gives $F_{disk_x} = 1281$ N and $F_{disk_y} = 896$ N. Therefore, $F_{disk} = 1563$ N and $\beta = 35°$.

c. The force F_{disk} is approximately 2.5 times as large as the weight of the person.

d. As β and θ are identical, the compressive force is the same as F_{disk}.

e. Lifting the weight corresponds to increasing the weight acting at the top of the spine a factor of two. This increases F_{disk} to 2312 N and the compressive force to 2311 N, as β changes to 34°.

f. Bending the knees to pick up an object allows the upper body to stay upright, i.e. brings the angle θ closer to 90°. For $\theta = 90°$, the change in the compressive force for picking up a 12 kg object is only 118 N, as opposed to more than 700 N in the example above.