

Solutions to Problem Set# 7

The solution is given for $g = 10.0\text{m/s}^2$. Of course you can use 9.8m/s^2 and get the same credit.

Problem 1) Y&F 6-50, p189

a) Since the force $F = Mg$ with total body mass M , the work done in doing a chin-up is

$$W = F\Delta h = M\text{kg} \times 10\text{m/s}^2 \times 0.40\text{m},$$

and work per kilogram is

$$W/M = 4.0\text{J/kg}$$

b) Let the mass of the muscles involved in doing a chin-up be m . From (a) we find an equation for the total work done in doing a chin-up

$$W = M \times 4.0\text{J/kg} = m \times 70\text{J/kg} \implies m/M = 4.0/70 = 5.7\%.$$

c) Now that the work per body mass is $W/M = 10\text{m/s}^2 \times \frac{1}{2} \times 0.40\text{m} = 2.0\text{J/kg}$, the ratio of mass of "used" muscle to total body mass is

$$m/M = 2.0/70 = 2.9\%.$$

d) Because children and adults have about the same percentage of muscle in their bodies, the problem depends on what percentage is needed in doing a chin-up. According to previous calculation a child needs a smaller percentage (2.9% in this problem) than an adult (5.7%). Thus children have advantage in doing chin-ups.

Problem 2) Y&F 6-54, p189

a) Following the usual procedure of decomposing the gravitational force into parallel ($mg \sin \theta$) and perpendicular ($mg \cos \theta$) components relative to the direction of the inclined plane, the normal force between package and the ramp is balanced by, and therefore equal to in magnitude, $mg \cos \theta$. So friction $f = N\mu_k = mg \cos \theta \mu_k = 15.2\text{N}$. We find that the direction of frictional force is opposite to the direction of the package's velocity, which means the angle between them is 180° . The work done by friction is

$$W_f = f \times 1.50\text{m} \cos 180^\circ = -15.2\text{N} \times 1.50\text{m} = -22.8\text{J}.$$

b) The package's height decreased by $1.50\text{m} \times \sin 12^\circ = 0.31\text{m}$. Work done by gravity is

$$W_g = 0.31\text{m} \times mg = 15.6\text{N}$$

c) Because it is perpendicular to the ramp at any instant, the normal force does no work on the package:

$$W_N = N \times 1.5\text{m} \times \cos 90^\circ = 0.$$

d) Total work $W_{tot} = W_f + W_g + W_n = -7.2\text{N}$. The work-kinetic energy theorem states that it is equal to change in the package's kinetic energy.

$$W_{tot} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \implies v_f = 1.4\text{m/s}$$

Problem 3 Y&F 6-60, p190

It is convenient to calculate the potential energy function first. Without loss of generality the zero point of potential is at infinity $V(\infty) = 0$.

$$V(x) = - \int_{\infty}^x \frac{\alpha}{x^2} dx = \frac{\alpha}{x}.$$

In this problem total energy is conserved but the form of energy can be transformed between kinetic and potential energies.

a) By conservation of energy

$$E_{tot}(x = 5.00m) = E_{tot}(x = 8.00 \times 10^{-10}m) \implies \frac{1}{2}mv_0^2 + \frac{\alpha}{5.00m} = \frac{1}{2}mv_1^2 + \frac{\alpha}{8.00 \times 10^{-10}m}$$

$$v_1 = \sqrt{v_0^2 + \left(\frac{\alpha}{5.00m} - \frac{\alpha}{8.00 \times 10^{-10}m}\right) / \left(\frac{m}{2}\right)} = 2.41 \times 10^5 m/s$$

b) At the closet position x_{min} the proton has all of its kinetic energy transformed into potential energy ($K = \frac{1}{2}mv^2 = 0$, $V(x)$ maximized).

$$E_{tot}(x = x_{min}) = \frac{\alpha}{x_{min}} = E_{tot}(x = 5.00m) \implies x_{min} = 2.82 \times 10^{-10}m.$$

b) When the distance between proton and nucleus restored, there is no total work done on the proton so its speed is the same as it's moving toward the nucleus, $3.00 \times 10^{-10}m/s$

Problem 4 Y&F 6-71, p191

The only difference from problem 3 is that now the potential function is the well-known

$$V(x) = \frac{1}{2}kx^2.$$

a) At maximum compression the kinetic energy vanishes, so all energy goes into potential form.

$$E_{tot}(x_f) = \frac{1}{2}x_f^2 = \frac{1}{2}mv_0^2 \implies x_f = 0.6m.$$

b) Again we need to solve the equation $E_{tot}(x_f) = \frac{1}{2}x_f^2 = \frac{1}{2}mv_0^2$, for v_0 this time, not for the given $x_f = 0.15m$. The solution for v_0 is

$$v_0 = 1.50m/s.$$