The purpose of these notes is to demonstrate the validity of the reciprocity theorem for the situation of Figure 7.20 (Page 278) by finding the flux Φ_{12} that a current in the inner ring creates through the outer ring.

Before beginning, note that Problem 7.20 uses the theorem to find the field at the outer ring. What is done in these notes is to use the field (borrowed from Chapter 11) due to the small ring to find the flux and in so doing show that the reciprocity theorem is valid in this situation. Note that for all *four* mentions (the derivation of Equation (41) on Page 278, Problem 7.20, the derivation of Equation (15) on Page 409 of Section 11.3, and these notes) the radius of the inner ring is taken to be much smaller than the radius of the outer ring.

Anyway, redraw your own version of Figure 7.20, but with the current in the inner ring and the field due to that current. All of the field lines will enclose the small ring, and if these lines do not enclose the outer ring, they will give no net contribution to the flux Φ_{12}. The net flux is then due to the field lines that intersect the common plane of the rings *inside* the inner loop but *outside* the outer loop.

Here's the argument (it's a handwave, but it is valid): The desired flux Φ_{12} is the integral of the field *outside* the outer loop. The point is, we know that field, while the field inside is difficult to integrate, to the point that we don't want to do it. Outside of the large ring, we can use the field as given by the approximation that we are far from the inner ring compared to its radius. By moving our integral outside of the ring, we get to use a field that we can find.

Anyhow, the field magnitude, as given by Equation (15) on Page 409, is

$$B_\theta = \frac{m}{r^3}\sin\theta = \frac{I_2 \pi R_2^2}{c r^3}$$

where $m = |\mathbf{m}| = I_2 \pi R_2^2 / c$ is the magnetic moment of the inner loop carrying a current I_2 and $\sin\theta = 1$ has been used for points in the plane of the loops (see Figure 11.6). Our flux is then

$$\Phi_{12} = \int B\,dA = \frac{\pi I_2 R_2^2}{c}\int_{R_1}^{\infty} \frac{2\pi r}{r^3}\,dr = \frac{2\pi^2 I_2 R_2^2}{c R_1}.$$

(There are two canceling minus signs that I've neglected; but then, I said I was finding a magnitude.)

Comparison of the above with Equation (40) on Page 278 shows that

$$\frac{\Phi_{21}}{I_1} = \frac{\Phi_{12}}{I_2}$$

and that is sufficient to demonstrate that for this model, $M_{12} = M_{21}$.