Supplemental Notes

It turns out that there is a very elegant way to get the position and magnitude of the image charge needed to find the field (potential) due to a charge Q near a grounded conducting sphere. Rather than start over with new notation, use what we had this morning (Monday November 6); The charge Q is at a position vector $\vec{r}_0$, the sphere has radius R and the image charge is $-q$ at a position $x\vec{r}_0$. We wish to find the magnitude of $-q$ and the value of x such that

$$\frac{Q}{|\vec{r}-\vec{r}_0|} = \frac{q}{|x\vec{r}_0-\vec{r}|}$$

when $\vec{r}$ is any vector with magnitude R. At this point, you are encouraged to create your own figure.

Anyway, this is what we do. We want to arrange the geometry so that the two denominators in the above expression will be proportional for any orientation of $\vec{r}$. This is done by having the triangle formed by $\vec{r}_0$ and $\vec{r}$ be similar to that formed by $\vec{r}$ and $x\vec{r}_0$. The common angle will be the same for any orientation of $\vec{r}$, so our condition for similarity of the triangles is

$$\frac{|\vec{r}|}{|\vec{r}_0|} = \frac{|x\vec{r}_0|}{|\vec{r}|} \quad \text{or} \quad x = \frac{R^2}{r_0^2}.$$

Then, $|\vec{r}-\vec{r}_0| = (r_0/R)\,|x\vec{r}_0-\vec{r}|$, and $q = Q(R/r_0)$ gives a potential of zero on the sphere. Neat, what?

Wednesday's quiz will not involve anything quite this difficult.