8.022 at ESG **Fall 2004**

Problem Set 03 **due Friday October 1**

Reading

Chapter 2 of Purcell, Sections 2.1-2.6

Problems from the text

- **2.1**
- **2.4**
- **2.5**
- **2.8**
- **2.10** This is a **keeper**; we'll be using it later on.
- **2.11**
- **2.12**

Even More With Dipole Fields

First off, make sure you've finished **Problem (B)** from **Problem Set 01** and **Problems (A) -(C)** from **Problem Set 02**; the following problems will use the same notation and some of the results.

(A) Start by showing that for a potential $\varphi_{\mathrm{a}} = A\,\varphi_1(r,\,\theta) = A\,r\,\cos\theta$ in spherical coordinates, with A a constant,

$$\mathbf{E}_{\mathrm{a}} \equiv -\boldsymbol{\nabla}\,\varphi_{\mathrm{a}}$$

is a constant. If this constant field is denoted $\mathbf{E}_{\mathrm{a}} = E_{\mathrm{a}z}\,\hat{\mathbf{z}}$, find $E_{\mathrm{a}z}$ in terms of A.

This perhaps peculiar use of notation allows for the possibility that $|\mathbf{E}| = -E_{\mathrm{a}z}$, as we're about to see. The subscript "a" in both $\mathbf{E}_{\mathrm{a}}$ and $E_{\mathrm{a}z}$ is for "ambient".

(B) With φ_{a} as defined above and φ_{d} as given in Problem Set 02, let $\varphi_{\mathrm{t}} = \varphi_{\mathrm{a}} + \varphi_{\mathrm{d}}$ be the sum of the fields (the subscript "t" is for "total"). Find A, and hence $\mathbf{E}_{\mathrm{a}}$ so that $\varphi_{\mathrm{t}} = 0$ at $r = R_0$, where R_0 is some constant radius. Your result for A should depend on p and R_0; recall from **Problem Set 02** that $\mathbf{p} = p\,\hat{\mathbf{z}}$, so that $p = p_z = |\mathbf{p}|$.

(C) Thus armed, find the total field $\mathbf{E}_{\mathrm{t}} = -\boldsymbol{\nabla}\varphi_{\mathrm{t}}$ and express your result in terms of $E_{\mathrm{a}z}$ and R_0 but NOT p; this will involve, in part, expressing $p = |\mathbf{p}|$ in terms of $E_{\mathrm{a}z}$. The signs may be tricky.

(D) If the energy densities $\frac{1}{8\pi}\mathbf{E}_{\mathrm{a}}^2$ or $\frac{1}{8\pi}\mathbf{E}_{\mathrm{t}}^2$ were integrated over all space exclusive of the interior of the sphere of radius R_0, the result would be inifite. If integrated over a region of finite size but with dimensions very large compared to R_0, the result would be proportional to the volume (convince yourself that this is true).

What we will do, then, is similar to what was done in Problem 1.35; we'll consider how the presence of the dipole field affects the total energy. That is, find

$$\iiint\limits_{V_{\mathrm{out}}} \frac{1}{8\pi}\left[\mathbf{E}_{\mathrm{a}}^2 - \mathbf{E}_{\mathrm{t}}^2\right]\, dv.$$

In the above integral, V_{out} is the volume outside of the sphere of radius R_0 centered at the origin, and the notation for the square of the magnitude of the electric field is Purcell's. In doing this integral, you are supposed to find that most of the nonzero parts have been done already (gee, I wonder where and by whom). One of the integrals may cause grief if you do the needed radial and angular integrals in one order. You can deal with it, just explain how and why. If you don't encounter grief, go look for it.

You may have noticed that by integrating over V_{out} we've avoided a nasty singularity at the origin. Why we can do this, and even why we're considering this at all, will be explained when we get to Chapter 3.