8.022 at ESG **Fall 2004**

Problem Set 04 **due Friday October 8**

Reading

Chapter 2 of Purcell, Sections 2.7–2.16

Optional: The Supplementary Notes **Coordinate-Free Representations of Vector Derivative Operators**, linked from our web page and sent separately.

Section **2.12** and **Figures 2.20 & 2.20′** are just great. The footnote on Page 68 applies to us as well. **Figure 2.31** on Page 78 gives the Cartesian Coordinate forms for the vector operators that we have seen before. The diagrams are fine, but for an extended version with gaudy color, the notes on *Leibniz' Rule*, from **18.02-ESG**, are linked from our web page.

Problems from the text:

Chapter 2: Problems 17, 19, 31, 32 (a mathematically rigorous "proof" is not needed).

(A)

Reminder: The fields considered here go back to the first problem set, when the scalar fields φ_1 and φ_{-2} were for spherical coordinates, not cylindrical. In this case, spherical coordinates make the math easier.

We don't want to get too far ahead in considering the dipole fields, so this will be simple. Using your expression, from **Problem Set 02, Problem (C)**, for the total field $\mathbf{E}_{\mathrm{t}}$, show that at $r = R_0$,

$$\mathbf{E}_{\mathrm{t}} = E_{\mathrm{n}}\,\hat{\mathbf{r}} = E_{\mathrm{n}}\,\hat{\mathbf{n}};$$

that is, $\mathbf{E}_{\mathrm{t}}$ is normal to the sphere of radius R_0, as it must be for the potential to be zero on the sphere.

(B)

Suppose a scalar field is given in spherical coordinates by

$$\psi\,(r,\,\theta) = r^l\,g_l(\theta), \qquad r > 0$$

for l and integer. Find (but do not solve. Yet.) the second-order differential that $g_l(\theta)$ must satisfy for $\nabla^2\psi = 0$. Show that for $l = 0$ or $l = -1$, $g_0 = g_{-1} =$ contant is a solution. Show that $g_1 = g_{-2} = \cos\theta$ is a solution. Show that in general that $g_l = g_{-(l+1)}$.