8.022 at ESG **Fall 2004**

Problem Set 05 **due Friday October 15**

Reading:

Study for the quiz instead.

Because of the quiz on Wednesday, this problem set will not include problems out of Purcell; there were enough of those on previous problem sets.

If you can do the following, you should be well-prepared for the math encountered on Wednesday's quiz.

(A)

Reminder: The fields considered here go back to the first problem set, when the scalar fields φ_1 and φ_{-2} were for spherical coordinates, not cylindrical. Direct reference is made to previous problem sets, which should have been done in spherical coordinates.

Recall the scalar electric potential for a dipole,

$$\varphi_{\rm d} = \frac{p\,\cos\theta}{r^2}, \qquad \mathbf{p} = (Qd)\hat{\mathbf{z}}, \qquad r \neq 0$$

(recall that "d" is a distance, not a differential or differential operator). One form of a quadrapole scalar field is obtained from

$$\varphi_{\rm q} = -\,(d\,\hat{\mathbf{z}})\cdot\boldsymbol{\nabla}\varphi_{\rm d}.$$

Find $\varphi_{\rm q}$ in terms of r, $\cos\theta$ and $Q_{zz} \equiv q\,d^2$. (Hint: You should have done much of the work already. Compare to Problem (B) of **Problem Set 04**.) Show by explicit calculation that

$$\nabla^2\varphi_{\rm q} = 0, \qquad r \neq 0.$$

Find $\mathbf{E}_{\rm q} = -\boldsymbol{\nabla}\varphi_{\rm q}$, but don't worry about putting into a nice form, yet.

NOTE: In the above, Q_{zz} is one component of the quadrapole moment, and hence has two indices. Other forms for quadrapole fields will not be considered quite yet. For now, just keep in mind that for the dipole field, the dipole moment $\mathbf{p}$ could be chosen to be in any direction, and we chose the z-direction. For a this form of the quadrapole field, we dotted the gradient of the dipole field into a displacement that was parallel to the dipole field. If we had not made this simple choice, we would have lots more to do.