8.022 at ESG **Fall 2004**

Problem Set 06 **due Friday October 22**

Reading:

Purcell Chapter 3

Problems from the text:

3.3: The hint is valid. If your integration is not "simple," try something else. You might even see what you get if the field lines leave the charge at an arbitrary angle.
3.5: Besides the calculation, make sure you understand the physics involved, in that two unnamed observers are claiming two different things. Whom do you support?
3.6: Should be straightforward. For those interested in a more mathematical treatment of what has become essentially a two-dimensional problem, see the notes **"Dipoles in Two Dimensions"**, linked from our page.
3.8: Try doing this for arbitrary parameters instead of the given 5 cm, 8 cm and 10 esu/m.
3.17: I get those numbers, so we'll hope they're correct.
3.20: The thing here is to decide which parameters involved in the geometry are kept fixed, and which vary. I've found two different ways that work, and there may be others.
3.22: In years past, a variation on this was a test question.
3.28: See comments for **3.6** above.

(A)

So far, we have constructed a scalar potential field $\varphi_{\mathrm{t}}(r,\,\theta)$ and the associated vector field $\mathbf{E}_{\mathrm{t}}(r,\,\theta)$, both in spherical coordinates. These fields have the properties that $\varphi_{\mathrm{t}} = 0$ for $r = R_0$, and you have shown that $\mathbf{E}_{\mathrm{t}} = E_n\hat{\mathbf{n}}$ at $r = R_0$; that is, the vector electric field is normal to the equipotential at $r = R_0$.

(1) Assume, then, that the sphere of radius R_0 is a conductor, and that $e = 0$ for $r < R_0$. Find an expression for $\sigma(\theta)$, the surface charge density on the sphere.

(2) Find the dipole moment due to this charge, given by

$$\int_{\text{sphere}} \mathbf{r}\, dq,$$

where $\mathbf{r}$ is, as usual, the vector from the origin to a point on the sphere. You will want to express $\mathbf{r}$ in tems of $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$, the angles θ and ϕ, and R_0. Hint: The result should not be surprising.