Problem 9.1
Wavelength of radio waves. (Giancoli 32-37.)

Channel 2:
\[ \lambda_2 = \frac{c}{f_2} = \frac{3.00 \times 10^8}{54.0 \times 10^6} = 5.56 \text{ m} \ . \]

Channel 69:
\[ \lambda_{69} = \frac{c}{f_{69}} = \frac{3.00 \times 10^8}{806 \times 10^6} = 0.372 \text{ m} \ . \]

Problem 9.2
Traveling electromagnetic waves.

The given electric field is in all three cases of the form
\[ \mathbf{E}(x, t) = \mathbf{E}_0 \sin(kx \pm \omega t + \alpha) \ , \]
with \( \mathbf{E}_0 \) perpendicular to the direction of propagation (the x-direction) and \( \alpha = 0 \) or \( \pi/2 \) (recall that \( \sin(\theta + \pi/2) = \cos \theta \)). For such a wave, the propagation direction is \( +\hat{x} \) if the argument is \( kx - \omega t + \alpha \) and \( -\hat{x} \) if the argument is \( kx + \omega t + \alpha \). \( k \) is the wavenumber, \( \lambda = 2\pi/k \) is the wavelength, \( f = \omega/2\pi \) is the frequency in Hertz, \( v = \omega/k \) is the speed, and \( n = c/v \) is the index of refraction. From the given expressions and these definitions, we can read off the answers to (a)–(e):

<table>
<thead>
<tr>
<th>prop. direct.</th>
<th>( \lambda ) (m)</th>
<th>( k ) (m(^{-1}))</th>
<th>( f ) (Hz)</th>
<th>( v ) (m/s)</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>case (1)</td>
<td>(-\hat{x})</td>
<td>4.00</td>
<td>1.57</td>
<td>7.50 \times 10^7</td>
<td>3.00 \times 10^8</td>
</tr>
<tr>
<td>case (2)</td>
<td>(+\hat{x})</td>
<td>2.00</td>
<td>3.14</td>
<td>1.50 \times 10^8</td>
<td>3.00 \times 10^8</td>
</tr>
<tr>
<td>case (3)</td>
<td>(-\hat{x})</td>
<td>1.00</td>
<td>6.28</td>
<td>2.13 \times 10^8</td>
<td>2.13 \times 10^8</td>
</tr>
</tbody>
</table>

(f) In order to construct the corresponding equations for \( \mathbf{B} \), we must remember two features of a traveling EM plane wave: (i) \( \mathbf{B} \) is in phase with \( \mathbf{E} \), and (ii) \( \mathbf{B} \) is perpendicular to both \( \mathbf{E} \) and the propagation direction such that \( \mathbf{E} \times \mathbf{B} \) points in the direction of propagation. If the vector \( \mathbf{k} \) indicates the direction of propagation, then our three cases must have the following orientations:
As for magnitudes: \( B = E/v = nE/c \). The full expressions for the magnetic fields of our three cases are thus (with \( B \) in Tesla)

- **case (1):** \( B_y = (-8.33 \times 10^{-8}) \sin(1.57x + 4.71 \times 10^8t) \), \( B_x = B_z = 0 \)
- **case (2):** \( B_z = (1.67 \times 10^{-7}) \cos(3.14x - 9.42 \times 10^8t) \), \( B_x = B_y = 0 \)
- **case (3):** \( B_y = (1.87 \times 10^{-7}) \cos(6.28x + 1.34 \times 10^9t) \), \( B_x = B_z = 0 \)

**(g)** The instantaneous Poynting vector for case (3) is (Giancoli Equation (32-18), p. 801):

\[
S = \frac{1}{\mu_0} (E \times B) = -\frac{1}{\mu_0} E_y B_y \hat{x} \\
= -\frac{(40)(1.87 \times 10^{-7})}{(4\pi \times 10^{-7})} \cos^2(6.28x + 1.34 \times 10^9t) \hat{x} \\
= (-6.0) \cos^2(6.28x + 1.34 \times 10^9t) \hat{x}
\]

The time average of \( \cos^2(A + Bt) \) is \( \frac{1}{2} \) for any \( A \) and \( B \), so the time-averaged Poynting vector for all positions (including the two specified) is

\[ \bar{S} = (-3.0)\hat{x} \]  
(units: Joules per square meter per second).

Thus we find that this traveling electromagnetic wave transmits energy in the \( -\hat{x} \) direction through space.

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**Problem 9.3**

*EM waves – Maxwell’s equations and the “speed of light”.*

We want to apply Faraday’s law to the given plane surface (area \( A_1 \)) and the rectangular loop that bounds it. For definiteness, we’ll take the normal to the surface to be in the \( +\hat{y} \) direction. To calculate \( \Phi_B \), we divide the surface up into many strips of thickness \( dz \) as shown in the diagram. Each strip will make a differential contribution to the flux of

\[
d\Phi_B = B_y dA = B_0 \cos(kz - \omega t) l \, dz
\]
The total flux will then be given by
\[
\Phi_B = \int d\Phi_B = B_0l \int_0^{\lambda/4} \cos(kz - \omega t) \, dz = \frac{B_0l}{k} [\sin(k\lambda/4 - \omega t) - \sin(-\omega t)] .
\]
Since \(k\lambda/4 = k(2\pi/k)/4 = \pi/2\), this becomes
\[
\Phi_B = \frac{B_0l}{k} [\cos(\omega t) + \sin(\omega t)] \quad \Rightarrow \quad \frac{d\Phi_B}{dt} = \frac{B_0l\omega}{k} [\sin(\omega t) - \cos(\omega t)] .
\]

Now to calculate \(\oint \mathbf{E} \cdot d\mathbf{l}\). Our choice of \(+\hat{y}\) (as opposed to \(-\hat{y}\)) for the normal to our surface dictates that our line integral be taken counterclockwise when viewed as in the diagram. Since \(\mathbf{E}\) is purely in the \(\hat{x}\) direction, \(\mathbf{E} \cdot d\mathbf{l} = 0\) along the top and bottom edges of the integration curve. This leaves us with
\[
\oint \mathbf{E} \cdot d\mathbf{l} = \int_0^l E_z(z = \lambda/4, t) \, dx + \int_0^0 E_x(z = 0, t) \, dx
\]
\[
= E_0 \sin(\omega t) \int_0^l dx + E_0 \cos(\omega t) \int_0^0 dx
\]
\[
= E_0l[\sin(\omega t) - \cos(\omega t)] .
\]
Faraday’s law asserts that \(\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B/dt\). For the case under consideration, this gives
\[
E_0l[\sin(\omega t) - \cos(\omega t)] = \frac{B_0l\omega}{k} [\sin(\omega t) - \cos(\omega t)] .
\]
This will be satisfied for all time only if \(E_0 = B_0\omega/k\). Given that \(c = \omega/k\) is the wave speed, we have the result \(B_0 = E_0/c\) as a consequence of Faraday’s law. Combining this with \(B_0 = \epsilon_0\mu_0 c E_0\) as obtained in lecture from Ampère’s law, we conclude that \(c = 1/\sqrt{\epsilon_0\mu_0}\) is the speed of light in vacuum.

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**Problem 9.4**

A standing electromagnetic wave.

(a) Any standing wave of the form \(\cos(kz) \cos(\omega t)\) has a wavelength of \(2\pi/k\) and a frequency in Hertz of \(\omega/2\pi\). For our wave, \(k = 2\sqrt{3} \text{ cm}^{-1}\) and \(\omega = 7.0 \times 10^{10} \text{ rad/s}\), so
\[
\lambda = 1.814 \text{ cm} , \quad f = 1.114 \times 10^{10} \text{ Hz} .
\]

(b) The index of refraction of the medium is
\[
n = \frac{c}{v} = \frac{c}{\omega/k} = \frac{(3.00 \times 10^{10} \text{ cm/s})}{(7.0 \times 10^{10} \text{ s}^{-1})/(2\sqrt{3} \text{ cm}^{-1})} = 1.48
\]

(c) To find \(\mathbf{B}\), we picture our \(\mathbf{E}\)-field as the linear superposition of two traveling waves, one traveling in the \(+\hat{z}\) direction and one in the \(-\hat{z}\) direction. Using the trigonometric identity
\[
2 \cos \alpha \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta) ,
\]
we can rewrite our E-field as

$$E = \frac{1}{2}E_0 \hat{x}[\cos(2\sqrt{3}z - 7.0 \times 10^{10}t) + \cos(2\sqrt{3}z + 7.0 \times 10^{10}t)] \ .$$

Now, using the rule discussed in problem 9.2(f), the B-field associated with the traveling wave propagating in the $+\hat{z}$ direction must point in the $+\hat{y}$ direction (assuming $E_0$ is positive) so that $\mathbf{E} \times \mathbf{B}$ points in the $+\hat{y}$ direction. Similarly, for the wave propagating in the $-\hat{z}$ direction, $\mathbf{B}$ must point in the $-\hat{y}$ direction. Thus our total $\mathbf{B}$-field must be

$$\mathbf{B} = \frac{1}{2}B_0 \hat{y}[\cos(2\sqrt{3}z - 7.0 \times 10^{10}t) - \cos(2\sqrt{3}z + 7.0 \times 10^{10}t)] \ .$$

Using the identity

$$2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta) \ ,$$

we can write this as

$$\mathbf{B} = B_0 \hat{y} \sin(2\sqrt{3}z) \sin(7.0 \times 10^{10}t) \ .$$

We see that in a standing wave, $\mathbf{B}$ is 90° out of phase relative to $\mathbf{E}$ both in space and in time. The value of $B_0$ is related to $E_0$ by

$$B_0 = \frac{k}{\omega}E_0 = \frac{E_0}{v} = \frac{nE_0}{c} \ .$$

(d) The instantaneous Poynting vector $\mathbf{S} = (\mathbf{E} \times \mathbf{B})/\mu_0$ for this wave at any point in space will have a time dependence of the form $\mathbf{S} \propto \sin(\omega t) \cos(\omega t)$. The average of $\sin(\omega t) \cos(\omega t)$ over one full period in time is zero, so $\mathbf{S} \equiv 0$ at all points. This result tells us that standing electromagnetic waves do not transmit energy through space. Compare this with the result of 9.2(g), where we found that a traveling electromagnetic wave does transmit energy.

**Problem 9.5**

Polarization of electromagnetic radiation.

(a) If we let $x = 0$, our electric fields vary with time as

(1) : $E_y = -E_0 \sin(\omega t)$ \quad $E_z = -4E_0 \sin(\omega t)$
(2) : $E_y = -E_0 \cos(\omega t)$ \quad $E_z = E_0 \sin(\omega t)$
(3) : $E_y = 2E_0 \sin(\omega t)$ \quad $E_z = 2E_0 \sin(\omega t)$

We can now plot a trace of $\mathbf{E}$ as a function of time at $x = 0$ and see the polarization easily (note: plots axes not to scale from one to the next):
(b) The amplitude of $\mathbf{B}$ is given by the electric field amplitude divided by $c$, since we are in a vacuum. We obtain the direction by requiring that $\mathbf{E} \times \mathbf{B}$ be in the direction of propagation. (It is helpful to note that, for $\mathbf{E}$ and $\mathbf{B}$ purely in the $y$-$z$ direction, $\mathbf{E} \times \mathbf{B} = \hat{z}(E_y B_z - E_z B_y)$.) Using these prescriptions, we find

\begin{align*}
(1) & : \quad B_y = (-4E_0/c) \sin(kx - \omega t) \quad B_z = (E_0/c) \sin(kx - \omega t) \\
(2) & : \quad B_y = (E_0/c) \sin(kx + \omega t) \quad B_z = (E_0/c) \cos(kx + \omega t) \\
(3) & : \quad B_y = (2E_0/c) \sin(kx - \omega t) \quad B_z = (2E_0/c) \cos(kx - \omega t + \pi/2)
\end{align*}

($B_z = 0$ for all cases.)

**Problem 9.6**

*Radiation pressure due to the sun.* (Giancoli 32-29.)

Let $P = 3.8 \times 10^{36}$ W be the Sun’s total power output. Assuming negligible absorption in the intervening space, the amount of energy per unit time crossing a spherical surface of radius $r$ centered on the Sun will also be $P$. The time-averaged Poynting flux (energy per unit area per unit time) at a distance $r$ from the center of the Sun will therefore be

$$\overline{S}(r) = \frac{P}{4\pi r^2} .$$

Assuming full absorption, the dust particles will feel a radiation pressure of $p_{\text{rad}} = \overline{S}/c$ (see Giancoli section 32-8, pp. 802-803). If the particles have a radius $a$, they will feel an outward (i.e. away from the Sun) force given by

$$F_{\text{rad}} = \pi a^2 p_{\text{rad}} = \pi a^2 \overline{S}/c = \frac{a^2 P}{4r^2c} .$$

The particles also feel a gravitational force directed towards the Sun. If $\rho$ is the mass density of the dust and $M$ is the Sun’s mass, the magnitude of this force is

$$F_G = \frac{G(\frac{4}{3} \pi a^3 \rho) M}{r^2} .$$
The magnitude of the radiation pressure force grows as $a^2$, while the magnitude of the gravitational force grows as $a^3$. So for very small particles, the outward radiation force should dominate, while for larger particles, the inward gravitational force will dominate. The scale is set by the particle size $a_0$ for which the two forces exactly balance one another:

$$\frac{a_0^2 P}{4r^2 c} = \frac{G(\frac{4}{3} \pi a_0^3 \rho) M}{r^2} \implies a_0 = \frac{3P}{16\pi G \rho MC}.$$ 

Plugging in the (many!) numbers, we get

$$a_0 = \frac{3(3.8 \times 10^{36})}{16\pi (6.67 \times 10^{-11})(2.0 \times 10^3)(1.99 \times 10^{30})(3.00 \times 10^8)} = 2.85 \times 10^{-7} \text{ m}.$$ 

Dust particles with a radius smaller than this would have been ejected by radiation pressure.

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**Problem 9.7**

*Snell’s law in action ⇒ dispersion! (Giancoli 33-46.)*

From Giancoli Figure 33-26 (p. 825), we can obtain approximate values for the index of refraction of silicate flint glass for the two wavelengths of interest:

$$\lambda_1 = 450 \text{ nm} : \ n_1 \simeq 1.64$$
$$\lambda_2 = 650 \text{ nm} : \ n_2 \simeq 1.62.$$ 

Now, consider either of the two rays. Define the angles $\alpha$, $\beta$, $\gamma$, and $\delta$ as shown in the diagram at right. Let $n$ be either $n_1$ or $n_2$ and $\theta$ be either $\theta_1$ or $\theta_2$. We’ll take the refractive index of the surrounding medium to be 1. Snell’s law (Giancoli Equation (33-5), p. 823) tells us that

$$\sin(45^\circ) = 1/\sqrt{2} = n \sin \alpha \quad \text{and} \quad n \sin \delta = \sin \theta.$$ 

Also, $\alpha = 90^\circ - \beta$ and $\delta = 90^\circ - \gamma$, so $\sin \alpha = \cos \beta$ and $\sin \delta = \cos \gamma$. Thus

$$1/\sqrt{2} = n \cos \beta \quad \text{and} \quad n \cos \gamma = \sin \theta.$$ 

Finally, we have $\beta + \gamma + 60^\circ = 180^\circ \Rightarrow \gamma = 120^\circ - \beta$. Solving for $\theta$ now gives

$$\theta = \arcsin(n \cos \gamma) = \arcsin[n \cos(120^\circ - \beta)]$$
$$= \arcsin \left[n \cos \left[120^\circ - \arccos \left(\frac{1}{\sqrt{2n}}\right)\right]\right].$$ 

For our two refractive indices of interest, this gives

$$\theta_1 = 68.1^\circ, \quad \theta_2 = 65.3^\circ.$$
Problem 9.8
Snell’s law in action ⇒ fiber optics! (Giancoli 33-53.)

The greatest test of our optic fiber’s ability to guarantee total internal reflection will occur when the beam entrance angle \( \alpha \to 90^\circ \). So, let’s consider that case in particular. Snell’s law gives

\[
\sin \alpha = \sin 90^\circ = 1 = n \sin \beta = n \cos \gamma.
\]

Now suppose that total internal reflection does not necessarily occur at point “a”. The angle \( \theta \) that the emerging beam makes with the normal to the fiber’s surface will be given by Snell’s law:

\[
\sin \theta = n \sin \gamma = n \sqrt{1 - \cos^2 \gamma}.
\]

Using \( n \cos \gamma = 1 \) from above, this becomes

\[
\sin \theta = n \sqrt{1 - 1/n^2} = \sqrt{n^2 - 1}.
\]

So \( \sin \theta \) increases as \( n \) gets bigger. \( \sin \theta = 1 \) (corresponding to \( \theta = 90^\circ \)) is the critical value for the onset of total internal reflection at point “a”. The condition on \( n \) for total internal reflection of all beams entering the fiber is therefore

\[
\sqrt{n^2 - 1} > 1 \quad \Rightarrow \quad n > \sqrt{2} \approx 1.42,
\]

where we have rounded up just to be safe.

END