MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
Physics II (8.022) – Prof. J. McGreevy – Spring 2008

Problem Set 1

Electric Charges and Electric Forces

Based on: Lecture 1 and your knowledge of vector calculus
Assigned reading: Purcell §1.1-1.8
Optional reading: Schey, chapter I

Due: Friday, February 8, 2007 at 5:00 PM in your section's lock box (located on the 3rd floor of Building 8, at the junction with Building 16). Be sure to write your name and section number on your pset. Please staple multiple pages together. Please check the course web site regularly for announcements.
`http://web.mit.edu/8.022/www`

The first few problems here (1-3) are meant to get everyone warmed up with the kind of math that we'll need to use frequently in 8.022. If you have trouble, the book *Div, Grad, Curl...* by Schey is designed to help. If the trouble is really serious, please consider 8.02.

1. **Taylor expansion.** [10 pts] Show that the function

$$f(x) = \ln\left(\frac{a_1 + x}{x}\right) + \ln\left(\frac{a_2 + x}{a_1 + a_2 + x}\right)$$

behaves like

$$f(x) \sim \frac{a_1 a_2}{x^2}$$

at large x. (Hint: Taylor expand in $u \equiv \frac{1}{x}$ around $u = 0$.)

2. **The gradient in various coordinate systems.** [20 pts] Review the definitions of Cartesian, cylindrical and spherical coordinates (see figure 1). Pick an arbitrary point P in 3-dimensional space and describe in each of these systems the following:

(a) the vector describing the position of P with respect to the origin.

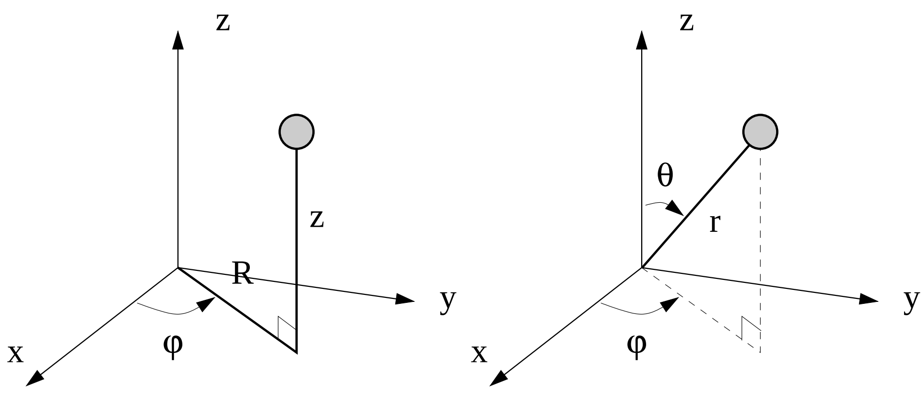

Figure 1: Our conventions for cylindrical coordinates (left) and spherical coordinates (right). Note that we measure the polar angle θ down from the z axis, and we use the symbol φ (pronounced 'curly phi' or 'varphi') for the azimuthal angle (reserving the symbol ϕ for the electrostatic potential).

(b) the distance element $d\vec{r}$ describing the change in location resulting from arbitrary increments of the coordinates.

(c) the infinitesimal volume element at point P.

(d) Use your answer to (b) to write the gradient $\vec{\nabla} f$ of a function f in the three coordinate systems. The gradient $\vec{\nabla} f$ is defined by demanding that in moving away from P an amount $d\vec{r}$, the infinitesimal change in the value of the function should be

$$df = \vec{\nabla} f \cdot d\vec{r}$$

in any coordinate system.

3. **Checking your answers to problem two.** [12 pts]

(a) Consider the function $f(x, y, z) = y$. Compute $\vec{\nabla} f$ in cartesian coordinates. Compute $\vec{\nabla} f$ in cylindrical coordinates. Verify that these computations produce the same vector. (If they don't, you may want to revisit problem 2.)

(b) Consider the function

$$r(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

(this function is the magnitude of the vector displacement from the origin). Compute $\vec{\nabla} r$ using both cartesian and spherical coordinates. Verify that these computations produce the same vector.

Finally, we get to talk about physics now. The rest of the problems are based on Lecture 1 and the reading from Purcell.

4. Purcell, problem 1.1: **Comparison of electric and gravitational forces.** [12 pts]

5. Purcell, problem 1.3: **Two charged volleyballs.** [12 pts]

6. Purcell, problem 1.4: **Charges on the corners of a square.** [12 pts] Note, *equilibrium* means that the charges will remain at rest if they are not disturbed. An equilibrium configuration is *stable* if the response to small disturbances is to return to the original configuration.

7. **Coulomb force between line charges.** [22 pts] Two thin rigid rods lie along the x-axis, as shown in Figure 1. Both rods are uniformly charged. Rod 1 has a length L_1 and a charge per unit length λ_1. Rod 2 has a length L_2 and a charge per unit length λ_2. The distance between the right end of rod 1 and the left end of rod 2 is L.

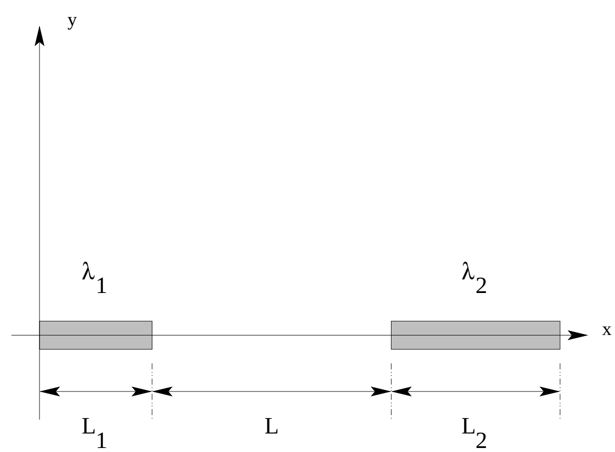

Figure 2: Coulomb force between line charges.

(a) Give an exact expression for the electrical force between the two rods, *i.e.* the force that one rod exerts on the other. If you get really stuck on the integral, you should always feel free to consult an integral table or try:

`http://integrals.wolfram.com`

(b) Use a first-order Taylor expansion to show that for $L_2 \gg L_1$, the electrical force on rod 1 is approximately

$$\vec{F}_1 = -\hat{x}\lambda_1\lambda_2 \ln\left(1 + \frac{L_1}{L}\right).$$

(c) Show that in the limit $L \gg L_1$ and $L \gg L_2$, your expression for the force between the rods reduce to the Coulomb force between two point charges. What are the magnitudes Q_1 and Q_2 of the point charges. *Hint: Look at problem 1 again.*