MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

Physics 8.022 ***Fall 2000***

8.022 Electricity and Magnetism

Quiz #2

2:05-3:35 PM, Monday, Nov. 13, 2000
Room 4-370

There are four problems worth a total of 100 points.
Answer all problems

This is a closed book quiz. No notes are allowed.
Calculators are not necessary.

In each problem, justify your answer.
Solutions with insufficient explanations will not be given full credit.

Useful Formulas

Resistors: $I = \int_S \vec{J} \cdot d\vec{a};\ \vec{J} = \sigma \vec{E};\ V = IR;\ P = VI = I^2 R = \frac{V^2}{R}$

Capacitors: $Q = CV;\ U = \frac{1}{2} CV^2 = \frac{1}{2}\frac{Q^2}{C}$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J}$$

$$\int_S \vec{E} \cdot d\vec{a} = 4\pi Q_{encl} \quad \oint_C \vec{E} \cdot d\vec{s} = -\frac{1}{c}\frac{d\Phi_B}{dt} \quad \int_S \vec{B} \cdot d\vec{a} = 0 \quad \oint_C \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} I_{encl}$$

Cyclotron motion: $\omega = \frac{qB}{\gamma mc}$ $\quad R = \frac{mv}{qB}$ Biot-Savart Law : $d\vec{B} = \frac{I d\vec{l} \times \hat{r}}{cr^2}$

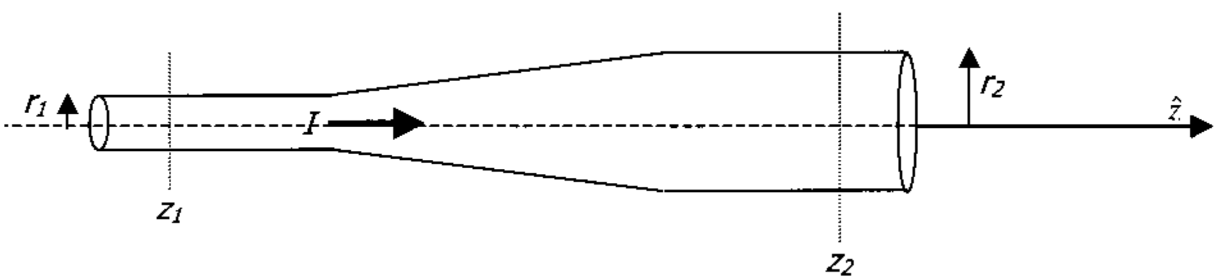
Lorentz Force: $\vec{F} = \vec{E} + q\frac{\vec{v}}{c} \times \vec{B}$

Transformation of fields: $\vec{E}'_\perp = \gamma \vec{E}_\perp + \gamma \vec{\beta} \times \vec{B}_\perp \quad \vec{E}'_\parallel = \vec{E}_\parallel$
$\vec{B}'_\perp = \gamma \vec{B}_\perp - \gamma \vec{\beta} \times \vec{E}_\perp \quad \vec{B}'_\parallel = \vec{B}_\parallel$

Lorentz trans. $x' = \gamma x - \gamma\beta ct \quad p' = \gamma p - \gamma\beta E / c$
$t' = \gamma t - \gamma\beta x / c \quad E' = \gamma E - \beta\gamma cp$ where $\beta = v / c$, $\gamma = 1/\sqrt{1-\beta^2}$

Problem 1

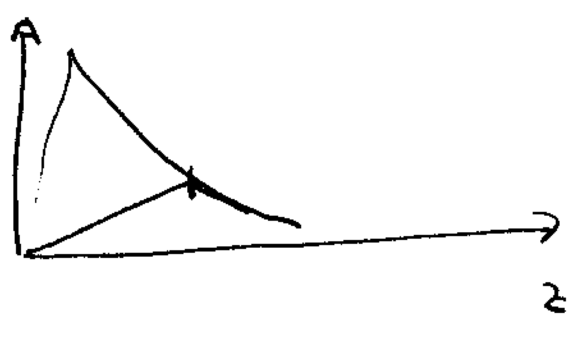
A current I flows in a wire which changes from radius r_1 to radius r_2 as shown below. The current density $\vec{J}$ inside the wire is uniform: $\vec{J} = \vec{J}(z)$. z_1 and z_2 are far from the place where the wire changes radius. Be sure to clearly state all assumptions and clearly state any symmetry arguments.



a) Find the current density $\vec{J}$ at z_1 and z_2 in terms of r_1, r_2, I, z_1 and z_2 and/or constants.

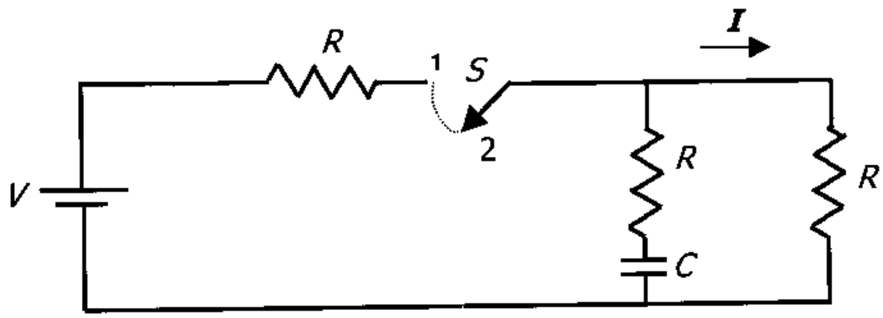
b) Find the magnetic field $\vec{B}$ at z_1 and z_2 both inside and outside of the wire in terms of r_1, r_2, I, z_1 and z_2 and/or constants.

c) One the same plot, sketch the magnetic field $\vec{B}(z_1, r)$ and $\vec{B}(z_2, r)$ as functions of r. Label the maximum value of the field in each case.



Problem 2

A circuit is connected as shown. At before *t=0*, the switch is in position 1 for a long time. At *t=0*, the switch *S* is moved to position 2 as shown.



a) Find the charge on the capacitor at *t=0*, $Q(0)$ in terms of *V*, *C* and/or *R* and constants.

b) Find the current at *t=0* after the switch is moved to position 2, $I(0)$ in terms of *V*, *C* and/or *R* and constants.

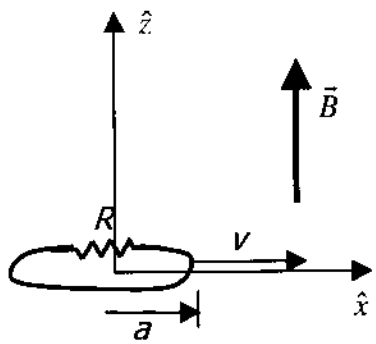
c) Find the current at *t>0* after the switch is moved to position 2, $I(t)$ in terms of *V*, *C*, *t* and/or *R* and constants.

d) Find the energy *U* stored in the capacitor at *t=0* in terms of *V*, *C* and/or *R* and constants.

e) Give the power dissipation $P(t)$ in the resistors and find the total energy dissipated $U = \int_0^\infty P(t)dt$ in terms of *V*, *C*, *t* and/or *R* and constants in each case.

Problem 3

A loop of mass *m*, radius *a* and area *A* with a resistor *R* moves with velocity $\vec{v} = v\hat{x}$ in a magnetic field $\vec{B} = B_o\left(\frac{x}{l}\right)\hat{z}$. *l>>a*, i.e. you may assume the magnetic field is constant over the area of the loop at any given time. At *t=0*, the loop is located at the origin.



a) Find the flux Φ through the loop at a function of position along the *x* axis in terms of *x, B_o, R, v, a, m* and/or A and constants.

b) Find the EMF $E(v)$ around the loop as a function of the velocity *v* in terms of *x, B_o, R, v, a, l, m* and/or A and constants.

c) Find the current flowing in the loop $I(v)$ as function of velocity *v*. Clearly indicate the direction of current flow. Use the results of the previous part to compute the power dissipated in the resistor *P(v)* as a function of the velocity in terms of *x, B_o, R, v, a, l, m* and/or A and constants.

d) Compute the kinetic energy of the loop as a function of time, *E(t)* in terms of *x, B_o, R, v, a, l, t, m* and/or A and constants . In performing the integration, remember $E = \frac{1}{2}mv^2$.

e) How far does the loop travel from the origin before it stops? How long does it take? Give your answer in terms of in terms of *x, B_o, R, v, a, l, m* and/or A and constants.

Problem 4

A particle of mass *m* and charge *q* is at rest in frame *F* at *t=0* in a constant applied electric field $\vec{E} = E_o\hat{x}$ and applied magnetic field $\vec{B} = B_o\hat{y}$ $E_o < B_o$. The particle is also viewed by an observer in frame *F'* moving with velocity $\vec{v}$ relative to *F*.

a) What must the direction of $\vec{v}$ be in order for there to be no applied electric field in frame *F*?

b) Find $\vec{v}$ such that there is no applied electric field in *F'* in terms of *m, q,* E_o and/or B_o and constants.

c) Find the magnetic field $\vec{B}'$ measured in *F'* in terms of *m, q,* E_o and/or B_o and constants.

d) Find the force acting on the particle $\vec{F}$ as measured by an observer in *F* in terms of *m, q,* E_o and/or B_o and constants.

e) Find the force acting on the particle $\vec{F}'$ as measured by an observer in *F'*. Show this is consistent with your answer for part d). Give your answer in terms of *m, q,* E_o and/or B_o and constants.

Note: In the statement of the problem, "applied" refers to all fields **except the field created by the point charge *q*.**