8.02x – Problem Set **1** Solutions

Problem 1 (5 points)

a) The ratio of the electric force to the gravitational force for an electron and a proton in a hydrogen atom is

$$\begin{aligned}
\left|\frac{F_e}{F_g}\right| &= \frac{k\frac{ee}{r^2}}{G\frac{m_e m_p}{r^2}} = \frac{k}{G}\frac{e^2}{m_e m_p} = \qquad (1)\\
&= \frac{9.0\times 10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2}{6.67\times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2}\frac{(1.6\times 10^{-19}\mathrm{C})^2}{(9.1\times 10^{-31}\mathrm{kg})(1.67\times 10^{-27}\mathrm{kg})} \approx 2\times 10^{39}.
\end{aligned}$$

b) In a), we have seen that the ratio of the forces does not depend on the distance, because both Newton's gravitational law and Coulomb's law have r^{-2} dependence. Therefore, the ratio F_e/F_g doesn't change with distance.

c) The universe is charge neutral on large scales. Therefore, there is no net electromagnetic force on astronomical distances. However, gravitational mass adds up, and gravitation is only an attractive (and long-distance) force, it dominates.

Problem 2 (5 points)

a) The ratio of the forces would be

$$\left|\frac{F_e}{F_g}\right| = \frac{k}{G}\frac{Q_E Q_M}{M_E M_M}. \qquad (2)$$

We need to estimate the number of protons (and electrons) in Earth and in the Moon. We assume protons contribute to roughly half the mass (neutrons are the other part, and electrons are 1000 times lighter). Therefore, the Earth has approximately $N_E \approx (M_E/m_p)/2$ protons. On the other hand, the Moon has approximately $N_M \approx (M_M/m_p)/2$ protons. Assuming the electrons' charge is $(1-10^{-11})$ times the proton charge, the Earth would carry a charge $Q_E \approx eN_E \times 10^{-11}$, and the moon would have charge $Q_M \approx eN_M \times 10^{-11}$. The ratio of the forces would then be

$$\begin{aligned}
\left|\frac{F_e}{F_g}\right| &= \frac{k}{G}\frac{\left(\frac{M_E}{2m_p}e\times 10^{-11}\right)\left(\frac{M_M}{2m_p}e\times 10^{-11}\right)}{M_E M_M} = \frac{k}{G}\frac{e^2}{4m_p^2}\times 10^{-22} = \qquad (3)\\
&= \frac{9.0\times 10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2}{6.67\times 10^{-11}\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2}\frac{(1.6\times 10^{-19}\mathrm{C})^2}{(1.67\times 10^{-27}\mathrm{kg})^2}\times 10^{-22} \approx 1\times 10^{14}.
\end{aligned}$$

b) This system would be highly unstable, the repulsive electrostatic force would tear them apart. It would be 14 orders higher than the gravitational attractive force!

Problem 3 (5 points)

a) The simplest example is placing another positive charge $Q_2 = Q_1$ symetrically at $x_2 = -x_1$. Another example is placing a bigger second charge with magnitude $Q_2 = 4Q_1$ at $x_2 = -2x_1$.

We can also derive a general rule for finding Q_2 and x_2. Let us assume the net force on charge Q_0 is zero. How do Q_2 and x_2 depend on Q_1 and x_1?

$$\begin{aligned} |F_{10}| &= |F_{20}|, \\ k\frac{Q_0 Q_1}{x_1^2} &= k\frac{Q_0 Q_2}{x_2^2}, \\ \frac{Q_1}{Q_2} &= \left(\frac{x_1}{x_2}\right)^2. \end{aligned} \tag{4}$$

b) When we displace Q_0 by Δx, its distance to charge Q_1 decreases, and so it is repelled more, also, it is repelled less from the other charge Q_2. Therefore it is pushed back into the equilibrium position – the charge will start to oscillate.

Y&F 21.9 (5 points)

The force of gravity must equal the electric force

$$F_g = F_e, \qquad \Rightarrow \qquad m_e g = k\frac{e^2}{r^2}, \qquad \Rightarrow$$

$$r = \sqrt{\frac{ke^2}{gm_e}} = \sqrt{\frac{(9.0\times 10^9 \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.602\times 10^{-19}\mathrm{C})^2}{(9.8\mathrm{m/s})(9.1\times 10^{-31}\mathrm{kg})}} \approx 5.1\mathrm{m}. \tag{5}$$

Y&F 21.25 (5 points)

The force on the α-particle is $\vec{F} = \vec{E}q = m\vec{a}$. We also know that its acceleration is such that in time t the particle stops. It was originally traveling to the right with velocity $\vec{v}_0$, and we want it to travel with velocity $\vec{v} = -\vec{v}_0$ in time t. From $\vec{v} = \vec{v}_0 + \vec{a}.t$ we have

$$\vec{v} = -\vec{v}_0 = \vec{v}_0 + \vec{a}.t \qquad \Rightarrow \qquad \vec{a} = -2\frac{\vec{v}_0}{t}.$$

Putting $\vec{a}$ into the equation for the force, we obtain

$$\begin{aligned} \vec{E}q &= m\vec{a} = -m\frac{2\vec{v}_0}{t} \\ \vec{E} &= -\frac{2m\vec{v}_0}{qt} = -\frac{2(6.64\times 10^{-27}\mathrm{kg})(1.5\times 10^3\mathrm{m/s}))}{2(1.602\times 10^{-19}\mathrm{C})(2.65\times 10^{-6}\mathrm{s})} \approx 23.5\mathrm{N/C}. \end{aligned} \tag{6}$$

<u>Y&F 21.31</u> (5 points)

a) The electric field doesn't change the electron's speed in the x-direction. So the time of travel before he flies out of the field is

$$t = s/v_x = 0.02\text{m}/1.6 \times 10^6\text{m/s} = 1.25 \times 10^{-8}\text{s}.$$

Because it just misses the upper plate, its distance traveled in the y direction is $h/2 = 0.5$cm. The motion in the y-direction is described by (the starting y-velocity is zero)

$$y = \frac{h}{2} = \frac{1}{2}a_y t^2 = \frac{1}{2}\frac{F_e}{m}t^2 = \frac{1}{2}\frac{Ee}{m}t^2,$$

so the magnitude of the electric field is then

$$E = \frac{h}{2}\frac{2m}{et^2} = \frac{(0.01\text{m})(9.1 \times 10^{-31}\text{kg})}{(1.602 \times 10^{-19}\text{C})(1.25 \times 10^{-8}\text{s})^2} \approx 364\text{N/C}. \tag{7}$$

b) If the particle was a proton, its mass would be greater, so it would accelerate less – and NOT hit the plates. Its vertical displacement would be (note that it will be downward, not upward, because the proton charge is positive!)

$$\begin{aligned} y_p &= \frac{1}{2}a_y t^2 = -\frac{1}{2}\frac{Ee}{m}t^2 = & (8) \\ &= -\frac{1}{2}\frac{(364\text{N/C})(1.602 \times 10^{-19}\text{C})}{1.67 \times 10^{-27}\text{kg}}(1.25 \times 10^{-8}\text{s})^2 \approx -2.73 \times 10^{-6}\text{m}. \end{aligned}$$

c) As in b), the proton will not hit the plates, because although it feels the same electric force (in magnitude), its mass is greater, so it accelerates less. Also, because its charge is opposite, its path will bend downwards, not upwards.

d) The acceleration of each particle in the electric field is $a = Ee/m$ is of the order 10^{10}m/s^2 for the proton (and 10^{13} for the electron). We can see it is much much greater than the gravitational acceleration g, and so it is reasonable to neglect gravity here.