MIT Department of Physics

Physics 8.02X **Spring 2005**

Solution to Problem Set #5

Problem 1 (10 points) Consider a simple parallel plate capacitor of Area A for each plate and separation d between the plates. The plates are given equal and opposite charges Q and the capacitor is isolated from the rest of the world

(a) What is the potential difference between the plates in terms of the given variables?

$$\Delta V = \frac{Qd}{\varepsilon_0 A}$$

(b) What is the stored energy in the capacitor?

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2(\varepsilon_0 A/d)} = \frac{Q^2 d}{2\varepsilon_0 A}$$

(c) Assume the plates are moved from a separation d to 2*d. How much does the stored energy in the capacitor change?

From the result of (b) we know that the new energy must be twice the old one, thus the energy change should be $\Delta U = 2U - U = U$.

(d) Show that the change in potential energy in question (c) is identical to the work done on the plates when moving them from d to 2*d.

Work done one the plate = –Work done by electric field = Word done against the field created by –Q: $\left(E_- = -\frac{Q}{2A\varepsilon_0}\right)$ for moving the plate with charge +Q:

$$\Delta U = -QE\_d = \frac{Q^2 d}{2A\varepsilon_0} = U$$

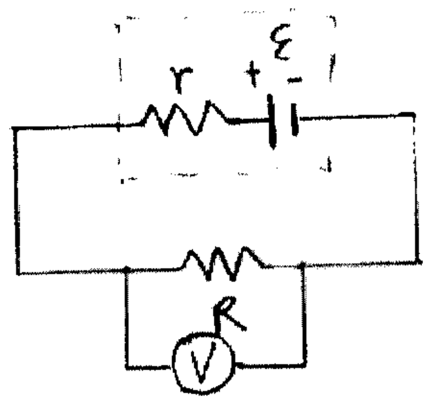
(e) Assume a dielectric with dielectric constant k=10 is inserted such that it fills the gap of the capacitor. How much does the stored energy change? How is energy conservation satisfied?

The old energy: $U = \frac{Q^2}{2C} = \frac{Q^2}{2(\varepsilon_0 A/d)} = \frac{Q^2 d}{2\varepsilon_0 A}$

The new energy: $U' = \frac{Q^2}{2KC} = \frac{Q^2}{2(K\varepsilon_0 A/d)} = \frac{Q^2 d}{2K\varepsilon_0 A}$

The energy conservation is still satisfied by noticing that the electric field did work on the dielectric: $W = U - U'$, which causes the reduce of stored energy in the capacitor.

Problem 2 (5 points) For the HVPS experiment, you found that the voltage across the output capacitor was lowest when the load had the lowest resistance. Explain this observation.



HVPS has an internal resistance r. Denote the resistance of the load by R:

$$V = IR = \frac{\varepsilon R}{R+r} = \frac{\varepsilon}{1+(r/R)} \Rightarrow \text{the lower the resistance } R\text{, the lower the voltage cross } R.$$

Problem 4 (5 points) Young&Freedman, page 972, Question Q25.11

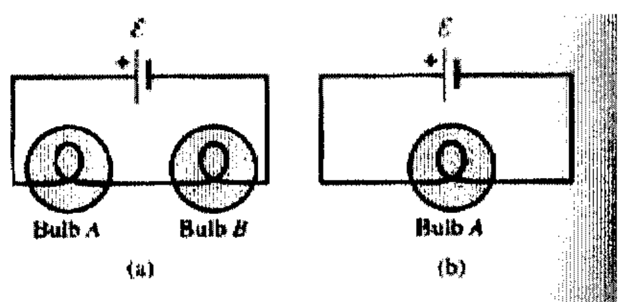


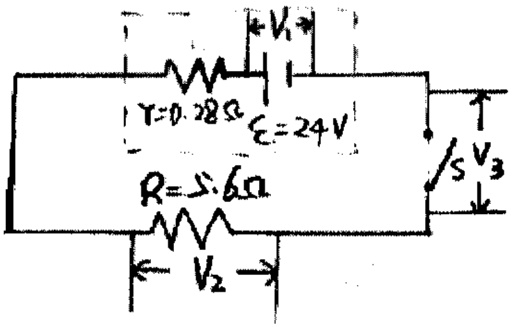
Figure 25.28 Question Q25.11.

Denote the resistance of the battery as r, and the resistance of Bulb A as R_A, the resistance of the battery as R_B. Since Bulb A and Bulb B are identical, we have $R_A = R_B = R$.

a) The current in the circuit is $I_a = \frac{\varepsilon}{r+2R}$, and the power dissipated in the Bulb A and Bulb B are the same $I_a^2 R$. Thus Bulb B should be as bright as Bulb A.

b) The current in the circuit is now $I_b = \frac{\varepsilon}{r+R} > I_a$, then the power dissipated in the Bulb A is now $I_b^2 R > I_a^2 R$, i.e. Bulb A is brighter than before.

Problem 5 (5 points) Young&Freedman, page 975, 25.34



When the switch is open,

a) $V_1 = 24.0\text{V}$; b) $V_2 = 0$; c) $V_3 = 24.0\text{V}$.

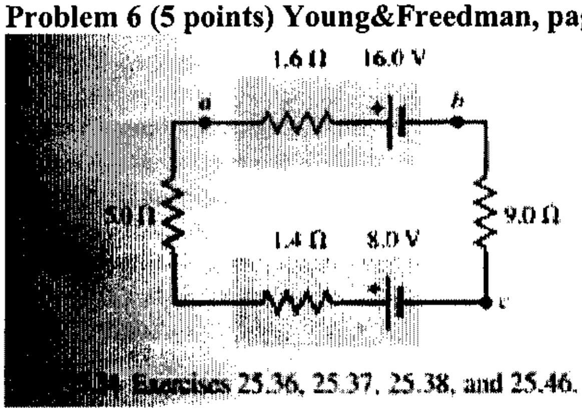
d) When the switch is closed,

a) $V_1 = \frac{\varepsilon R}{R+r} = \frac{24.0\text{V} \times 0.28}{5.6+0.28} = 22.86\text{V}$;

b) $V_2 = V_1 = \frac{\varepsilon R}{R+r} = \frac{24.0\text{V} \times 5.6}{5.6+0.28} = 22.86\text{V}$;

c) $V_3 = 0$.

Problem 6 (5 points) Young&Freedman, page 975, 25.38



The net emf of the circuit: $\varepsilon = 16\text{V}-8\text{V}=8\text{V}$

a) $I = \frac{V_{cb}}{9.0\Omega} = \frac{1.9\text{V}}{9.0\Omega} = 0.21\text{A}$

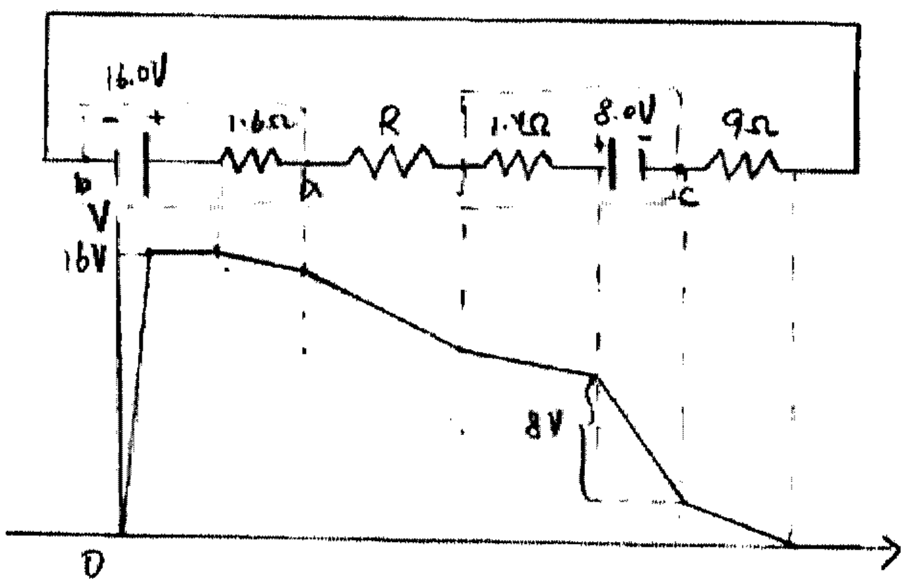
b) The total resistance of the circuit

$$R_t = \frac{\varepsilon}{I} = \frac{8\text{V}}{0.21\text{A}} = 37.89\Omega$$

Then the unknown resistance is

$$R = R_t - 1.6\Omega - 1.4\Omega - 9\Omega = 37.89\Omega - 1.6\Omega - 1.4\Omega - 9\Omega = 25.89\Omega$$

c)

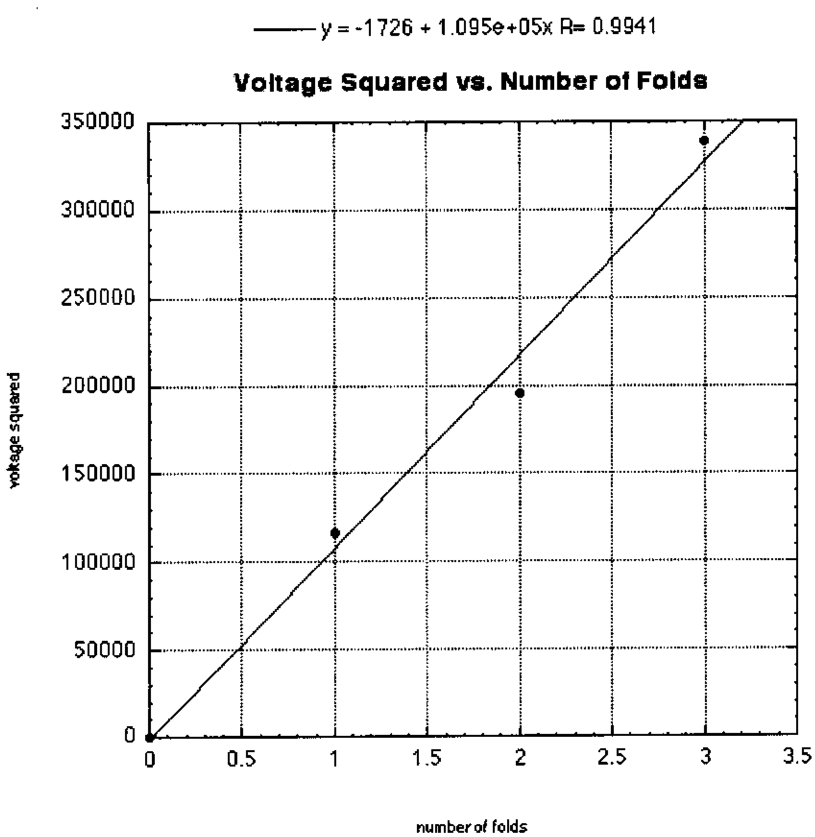


Massachusetts Institute of Technology
8.02X Electricity and Magnetism

Problem One (Electrostatic Force Experiment):

My data for Experiment EF is contained in Table 1. The graph of the average voltage squared vs. number of folds is shown in Figure 1.

trial	one fold (volts)	two folds (volts)	three folds (volts)	number of folds	Ave voltage squared (volts^2)
0	0	0	0	0	0
1	380	410	590	1	116281
2	300	500	560	2	195069
3	370	430	560	3	338724
4	345	450	630		
5	310	410	570		
6	no data	450	no data		
ave	341	442	582		



1. I used a linear regression to find the slope and intercept for the best-fit straight line for the graph of the experimental values of V^2 *vs.* n. My result

$$V^2 = -1725.5 + 1.095 \times 10^5 n .$$

2. Since the slope for our idealized washers is given by

$$slope = \frac{V^2}{n} = \frac{\rho t g 2 d^2}{\varepsilon_0}$$

where:

- thickness of perf-board + tape, $d = 1.7 \times 10^{-3} m$;
- thickness of Aluminum foil, $t = 7.6 \times 10^{-6} m$;
- density of Aluminum foil, $\rho = 2.7 \times 10^3 kg/m^3$;
- acceleration due to gravity, $g = 9.8 m/s^2$.

We can solve for the permittivity of free space ε_0

$$\varepsilon_0 = \frac{\rho t g 2 d^2}{slope} = \frac{2(2.7 \times 10^3 kg/m^3)(7.6 \times 10^{-6} m)(9.8 m/s^2)(1.7 \times 10^{-3} m)^2}{(1.095 \times 10^5)} = 1.06 \times 10^{-11} C^2/N-m^2$$

The accepted experimental value for permittivity of free space ε_0 is

$$(\varepsilon_0)_{expt} = 8.85 \times 10^{-12} C^2/N-m^2$$

The fractional difference between my result and the accepted value is

$$f = \frac{\varepsilon_0 - (\varepsilon_0)_{expt}}{(\varepsilon_0)_{expt}} = \frac{(1.06 \times 10^{-11} - 8.85 \times 10^{-12}) C^2/N-m^2}{8.85 \times 10^{-12} C^2/N-m^2} = 0.20$$

The principal sources of measurement error are in the measurements of the thickness between the washers and the thickness of the aluminum foil. In additional, when taking data any unaccounted surface tension due to the contamination of the surface with grease will affect the forces required to lift the foil. Also errors introduced in the folding will cause small errors in the comparison of the results for different folds. Finally our idealized model introduces a theoretical error. Our result is still within 20% of the accepted value and that is pretty good for this type of experiment!

Problem 2: (Electrostatic Force experiment):

a) Using Gauss' Law, find an expression for the electric field between two discs of radius R that are separated by a distance d. The discs have opposite charges that are equal in magnitude placed on them. You may neglect edge effects. Make a sketch of the electric filed lines when you include edge effects.

Answer: If we ignore edge effects the electric field is due to two 'infinite planes' of opposite surface charge density $+\sigma$ and $-\sigma$.

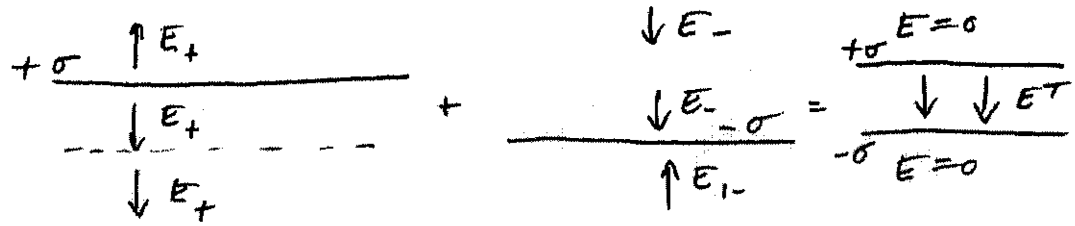


Figure 2: Superposition of two 'infinite planes' of opposite surface charge density $+\sigma$ and $-\sigma$.

In figure 2 we show the electric fields due to each sheet and their superposition. Outside the plates the fields cancel and inside the plates the fields add up to give the total field. Since each plate contributes the same magnitude of field, the total electric field is

$$E^{total} = E_+ + E_- = 2E_+$$

We can calculate the electric field due to an infinite positive plate with surface charge density $+\sigma$ using Gauss' Law. Figure 3 shows our choice of Gaussian surface.

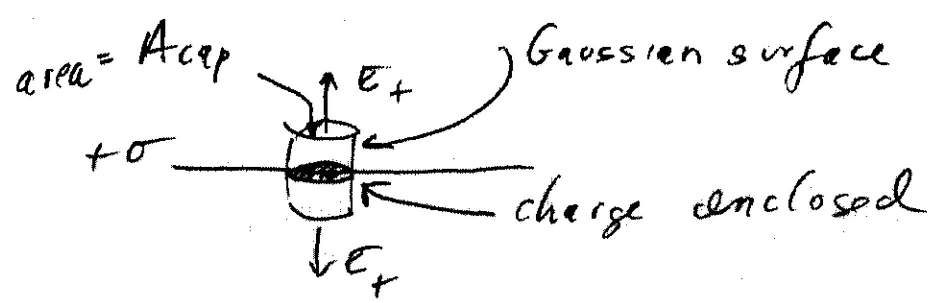


Figure 3: Gaussian surface for infinite positive plate with surface charge density $+\sigma$

Then Gauss' Law

$$\oint_{\substack{closed\,surface\\S}} \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

becomes

$$E_+ A_{cap} + E_+ A_{cap} = \frac{\sigma A_{cap}}{\varepsilon_0}.$$

Thus $E_+ = \dfrac{\sigma}{2\varepsilon_0}$. So the total electric field between the plates is

$$E^{total} = \frac{\sigma}{\varepsilon_0}.$$

We have assumed that the surface charge density is uniform and is therefore equal to

$$\sigma = \frac{Q}{\pi R^2}$$

In figure 4 we show the field lines when we no longer ignore the edge effects.

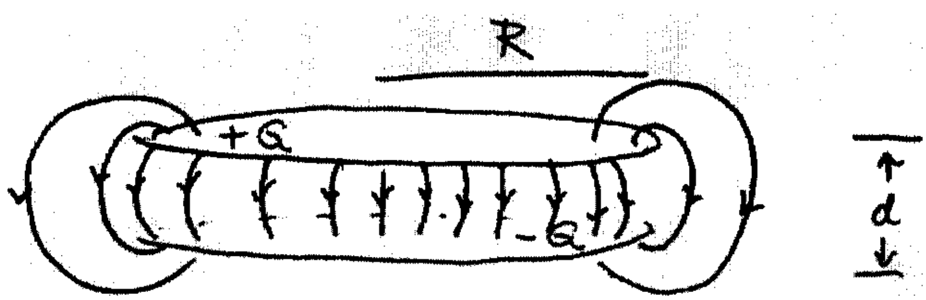


Figure 4: Electric field including edge effects for rings

b) Suppose a voltage difference ΔV is applied across the two discs. Show that the charge on the positive plate is given by the expression, $Q = \dfrac{\varepsilon_o \pi R^2 \Delta V}{d}$ when you ignore edge effects.

Answer: The voltage difference between the plates is

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{r} = E^{total} d = \frac{Qd}{\pi R^2 \varepsilon_0}$$

So the charge is given by

$$Q = \frac{\varepsilon_o \pi R^2 \Delta V}{d}.$$

c) Taking into consideration edge effects, the actual expression for the charge is given by $Q = f\frac{\varepsilon_0 \pi R^2 \Delta V}{d}$ where f is a correction factor that depends on the ration of separation to plate radius. For circular plates, the factor f depends on d/R as follows:

d/R	f
0.2	1.286
0.1	1.167
0.05	1.094
0.02	1.042
0.01	1.023

Explain why the factor f is always greater than 1. Where is the 'extra charge'?

Answer: If we take into account edge effects the charge increases by a factor f

$$Q = f\frac{\varepsilon_0 \pi R^2 \Delta V}{d}.$$

As $d/R \to 0$ the factor f approaches unity corresponding to our case where we approximate the discs as infinite parallel plates. The factor f is always greater than one because the effective area available for the charge to reside on is larger including the outside surfaces of the discs. (See Figure 4.)

It turns out that the electric field on the axis passing through the center of the discs varies unlike our assumption of a constant field. In fact an exact calculation for the electric field on the center axis midway between the plates yields

$$E_z = \frac{Q}{\pi R^2 \varepsilon_0}(1 - \frac{(d/2)/R}{(1 + ((d/2)/R)^2)}).$$

In the limit as $d/2)/R \to 0$ this reduces to

$$E_z \cong \frac{Q}{\pi R^2 \varepsilon_0}(1 - (d/2)/R)).$$

This field is smaller than our constant field

$$E^{total} = \frac{Q}{\pi R^2 \varepsilon_0}.$$

Now the voltage difference is fixed and is always equal to

$$\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}.$$

Therefore if the electric field is smaller at the midpoint it must be greater at the surface of the plates. The relationship between the surface charge density and the normal electric field on the surface of a conductor is always

$$\frac{\sigma}{\varepsilon_0} = E_{normal}.$$

Therefore if the electric field is bigger on the surface so is the charge density. This gives a second indication that the physical discs can hold more charge than the idealized model for the discs.

d) Find an expression for the capacitance of the circular discs neglecting edge effects. Now take into consideration the correction factor f. Is the actual capacitance larger or smaller?

Answer: If we ignore edge effects, the capacitance is given by

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 \pi R^2}{d}.$$

If we include edge effects, the capacitance is

$$C = \frac{Q}{\Delta V} = f\frac{\varepsilon_0 \pi R^2}{d}$$

e) How would you expect your results in parts a) – d) to change if you used two washers of inner radius R_1 and outer radius R_2 instead of two discs?

Answer: If we used two washers instead of discs there is less area available for the charge so we should expect the capacitance to decrease.