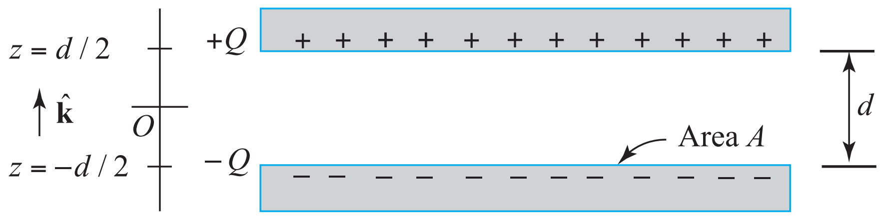
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics: 8.02

Capacitance of a Parallel Plate Capacitor

A parallel plate capacitor consists of two long parallel conducting plates each of area A that are separated by a distance d. The upper plate is positively charged with charge $+Q$ and the lower plate is negatively charged with charge $-Q$. Choose a coordinate system such that plates are lying in the *xy*-plane with the upper plate located at $z = d/2$ and the lower plate located at $z = -d/2$. You may assume that the charge distributions on the plates are uniform and neglect edge effects.

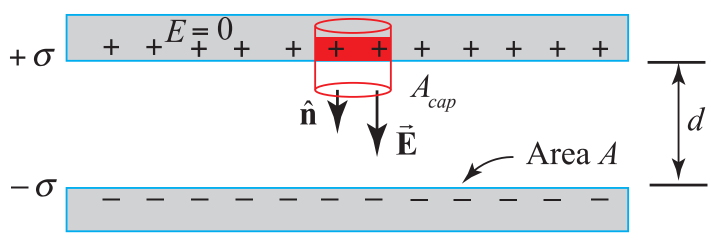


cross sectional view of two very lon and wide parallel plates

Calculate the capacitance of this capacitor.

Solution:

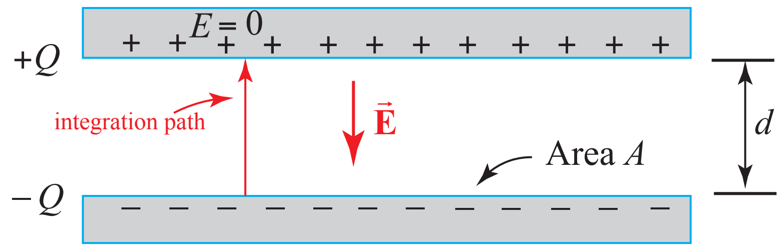
The electric field inside the conducting plates is zero so you can choose a Gaussian surface with one end-cap between the plates, and the other end-cap inside the upper positive plate as shown in the figure below. The charge density on the positive plate is $\sigma = Q/A$.



Now apply Gauss's Law:

$EA_{cap} = \dfrac{\sigma A_{cap}}{\varepsilon_0}$, therefore $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$. Because $\sigma > 0$ at ($z = d/2$), the electric field $\vec{\mathbf{E}}$ points in the negative $\hat{\mathbf{k}}$ -direction Therefore in vector notation, we have

$$\vec{\mathbf{E}} = \begin{cases} \vec{\mathbf{0}}, & z > d/2 \\ -\dfrac{Q}{A\varepsilon_0}\hat{\mathbf{k}}, & -d/2 < z < d/2 \\ \vec{\mathbf{0}}, & z < -d/2 \end{cases}$$



The electric potential difference between the plates is then

$$V(d/2)-V(-d/2)=-\int_{z=-d/2}^{z=d/2}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}=-\int_{z=-d/2}^{z=d/2}\left(-\frac{Q}{A\varepsilon_0}\hat{\mathbf{k}}\right)\cdot dz\hat{\mathbf{k}}$$

$$=\frac{Q}{A\varepsilon_0}\int_{z=-d/2}^{z=d/2}dz=\frac{Q}{A\varepsilon_0}((d/2)-(-d/2)=\frac{Qd}{A\varepsilon_0}$$

The capacitance is defines to be

$$C=\frac{Q}{\left|\Delta V\right|}=\frac{A\varepsilon_0}{d}.$$

Note that the capacitance is a property of the geometrical configuration of the plates and does not depend on the charge on the plates. The amount of charge that appears on the positive plate depends on the potential difference across the plates $Q=C\left|\Delta V\right|=\dfrac{A\varepsilon_0}{d}\left|\Delta V\right|$.