Exploration 2
Helmholtz Coil

Force and Torque on Magnetic Dipole

W07D2

Announcements

Week 7:

Week 7 LS 3 and LS 4 due Tuesday at 11 pm

Problem Set 6 Due Wednesday at 11 pm

Resistor Piano Experiment due Friday at 11

No Class on Friday

Week 8: Spring Break. Enjoy.

Week 9:

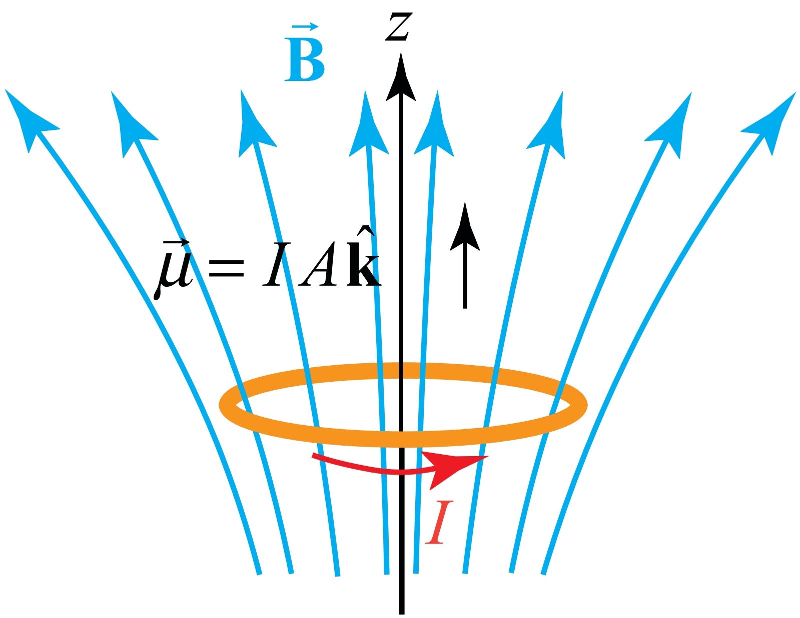
Week 9 LS 1 and LS 2 due Sunday at 11 pm

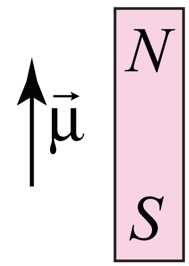
Week 9 LS 3 and LS 4 due Tuesday at 11 pm

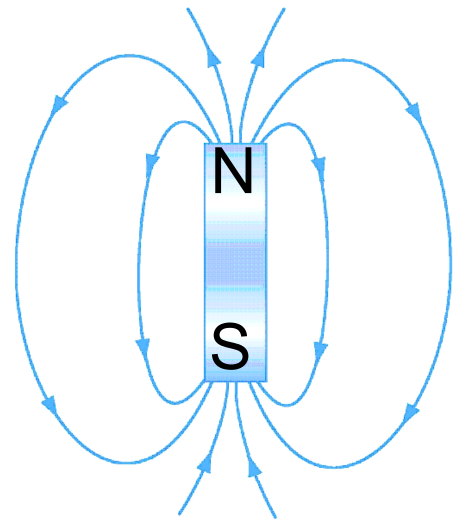
Problem Set 7 Due Wednesday at 11 pm

Magnetic Force on a Dipole in a Non-Uniform Magnetic Field

Force on Magnetic Dipole







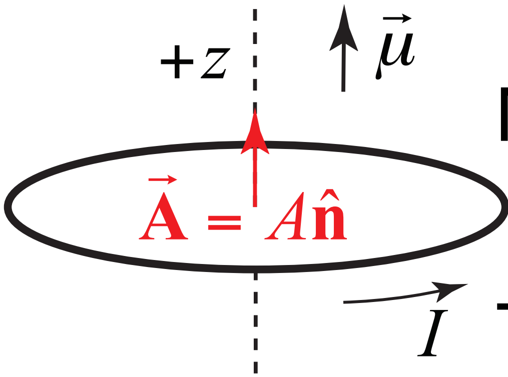
Special case: Along z-axis, for current loops, the magnetic field points along z-axis and the force on a dipole situated on the z-axis is

$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

Dipole is attracted to region of greater field strength

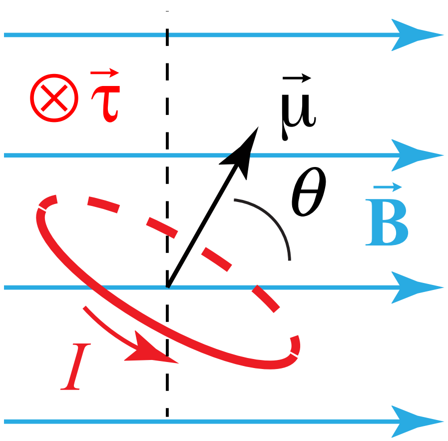
Summary: Magnetic Dipoles

No Magnetic Point Charges: $\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$



Magnetic Dipole Moment: $\vec{\mu} \equiv IA\,\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$

Torque: $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}_{\text{ext}}$



Potential Energy: $U(\boldsymbol{\theta}) = -\vec{\mu} \cdot \vec{\mathbf{B}}_{\text{ext}}$

Force: $\vec{\mathbf{F}} = -\vec{\nabla}U = \vec{\nabla}(\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}_{\text{ext}})$

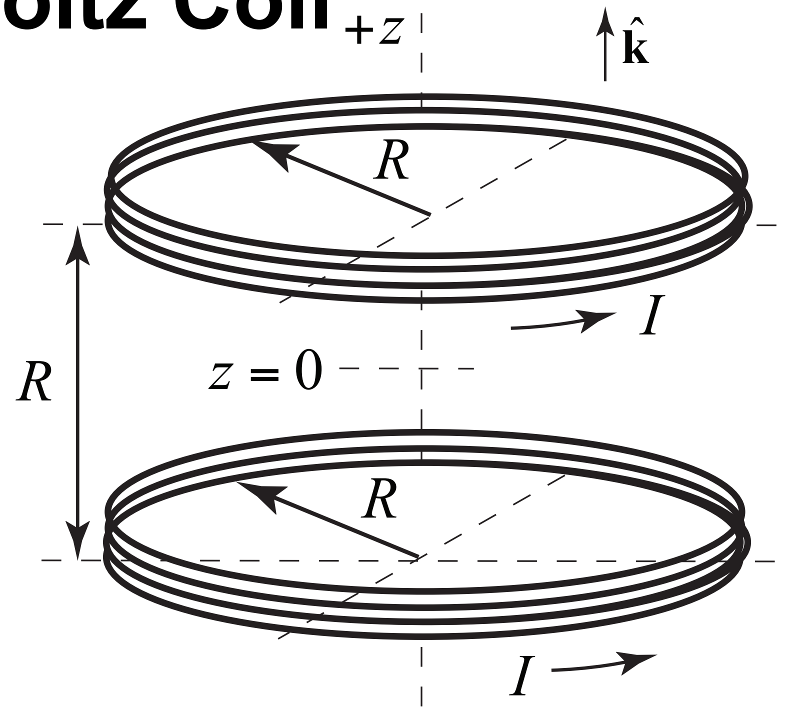
Special case: $\vec{\mu} = \mu_z \hat{\mathbf{k}} \Rightarrow F_z = \mu_z \dfrac{\partial B_z}{\partial z}$

Demo: Exploration 2: Magnetic Dipole in Helmholtz Coil

The experimental apparatus consists of two coils connected to a power supply along with a tube in which a small dipole magnet hangs from a spring that can be moved by a rod. The coils can be connected to a power supply that produces a current in the coils that creates a magnetic field.

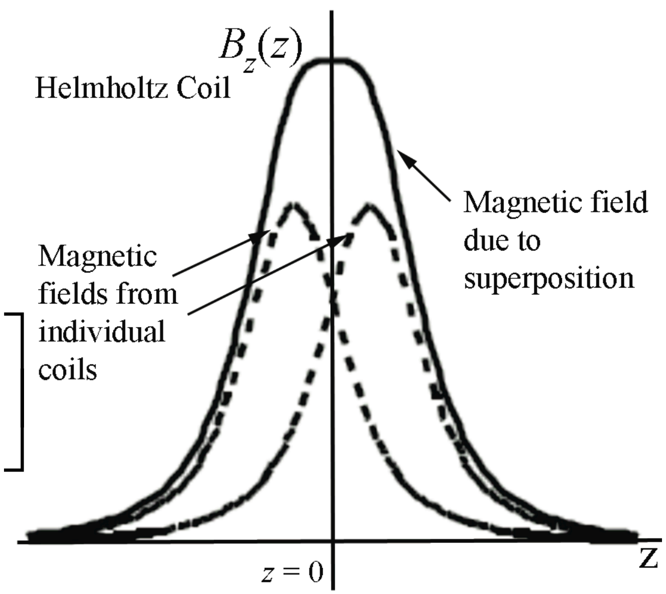
Magnetic field of a Helmholtz Coil

Consider two N-turn circular coils of radius R, each perpendicular to the axis of symmetry, with their centers located at z = 0. There is a steady current I flowing in the same direction around each coil. The magnetic field on the symmetry axis at a distance z > 0 from the midpoint between the centers of the two coils is the superposition of the fields of the two coils. If we approximate the field of each coil as the sum of N rings situated on the planes $z = \pm R/2$ then the z-component of the magnetic field is

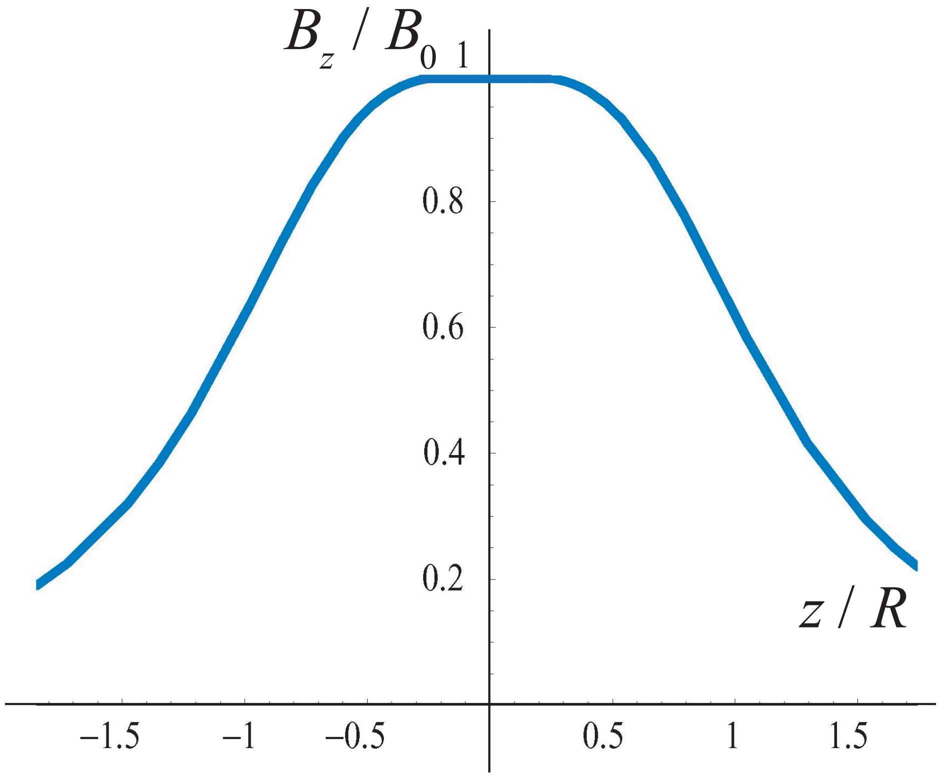


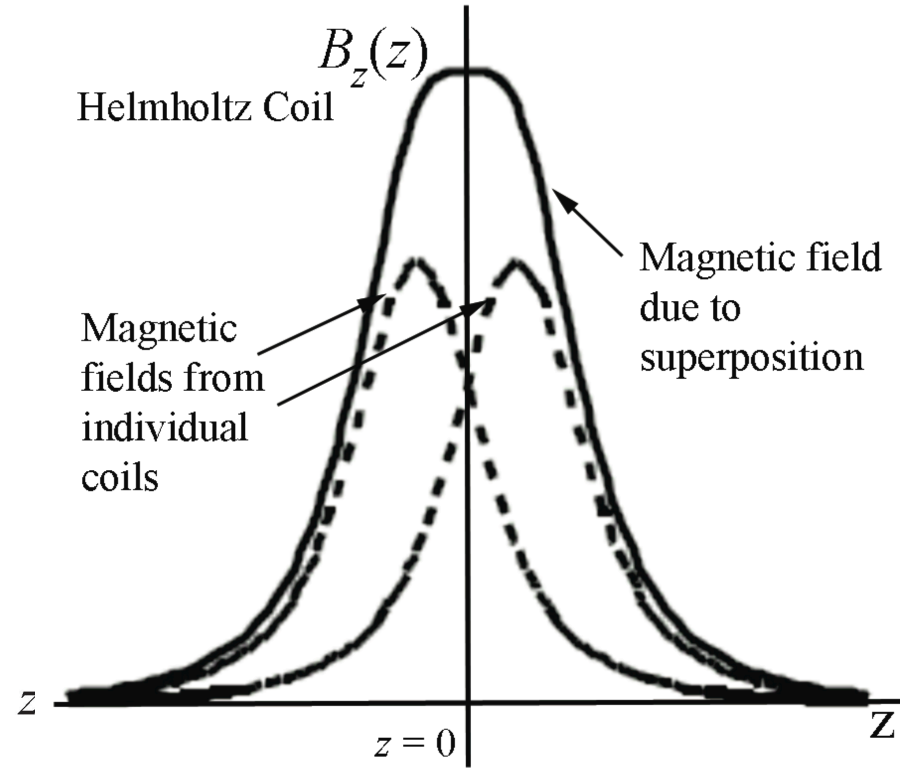
$$B_z(z) = B_{z,\text{top}}(z) + B_{z,\text{bot}}(z)$$

$$= \frac{\mu_0 N I R^2}{2}\left[\frac{1}{[(z-R/2)^2+R^2]^{3/2}} + \frac{1}{[(z+R/2)^2+R^2]^{3/2}}\right]$$



Magnetic field of a Helmholtz Coil





The magnetic field at the center of the coil is

$$B_z(0) = \frac{\mu_0 NI}{(5/4)^{3/2} R}$$

A plot of $B_z(z)/B_z(0)$ is shown in the figure above left

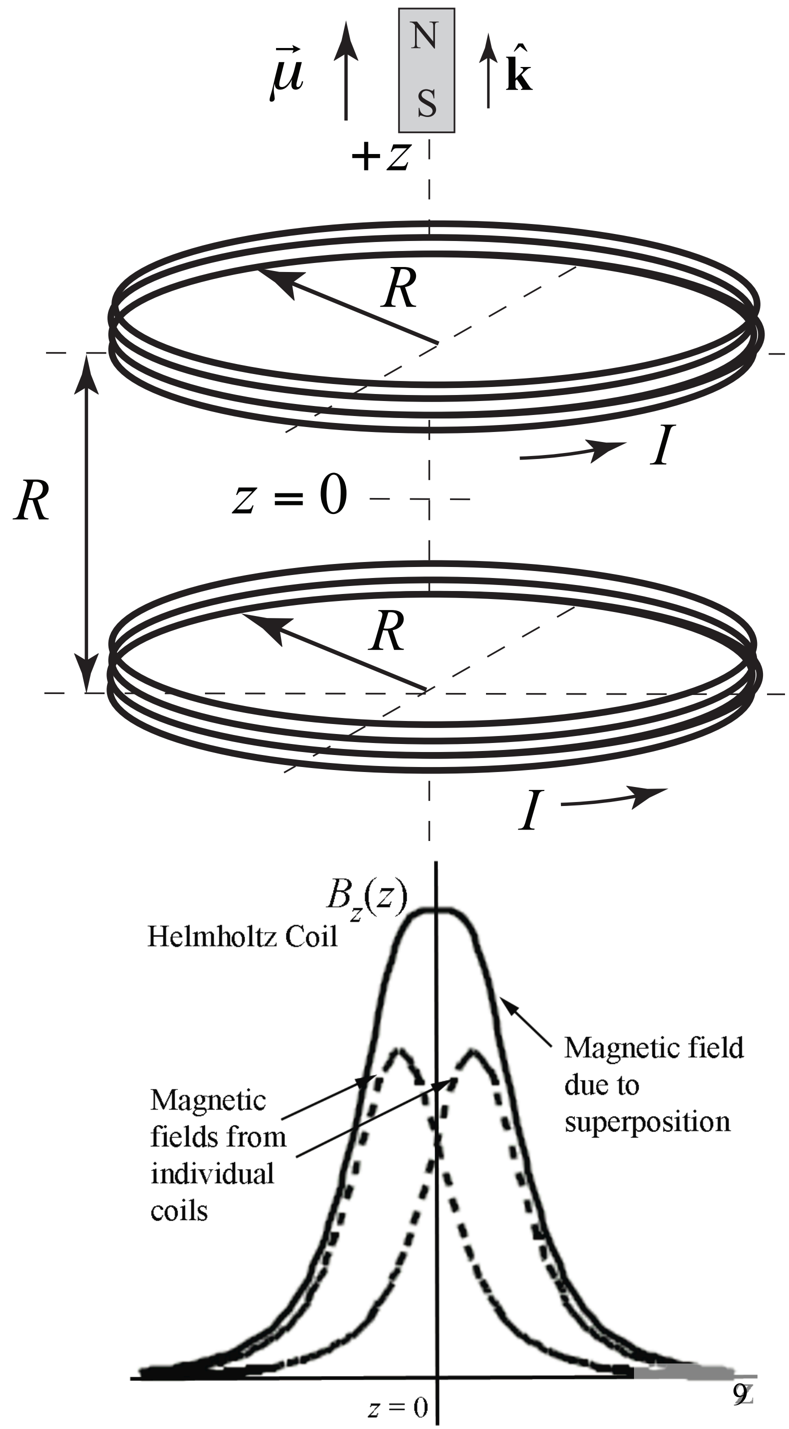
$$B_z(z)/B_z(0) = \frac{(5/4)^{3/2} R^3}{2} \left[\frac{1}{[(z - R/2)^2 + R^2]^{3/2}} + \frac{1}{[(z + R/2)^2 + R^2]^{3/2}} \right]$$

CQ.: Magnetic Dipole in Helmholtz Coil

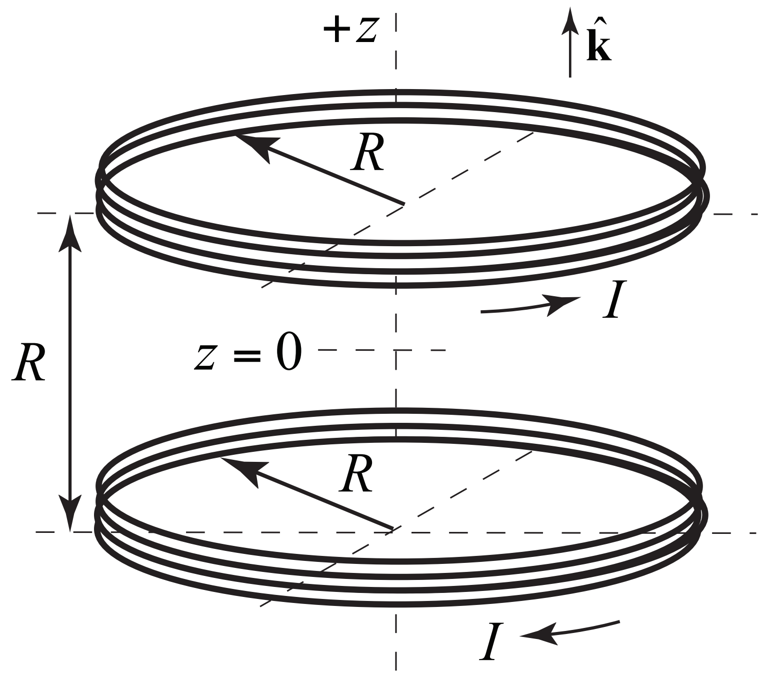
A fixed dipole is located on the symmetry axis above the two coils ($z > R/2$), and is pointing in the positive z-direction. Which of the following statements is true? Recall the force on a dipole is

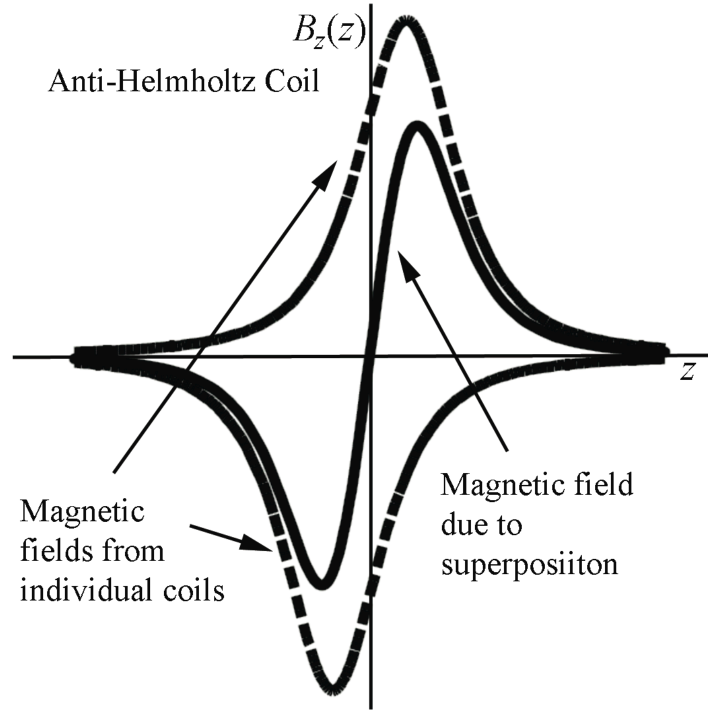
$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

1. The dipole experiences a force in the positive z-direction but zero torque.
2. The dipole experiences a force in the negative z-direction but zero torque.
3. The dipole experiences neither force nor torque.
4. The dipole experiences a force in the negative z-direction and a non-zero torque.
5. The dipole experiences a force in the positive z-direction and a non-zero torque.



Magnetic field of a Anti- Helmholtz Coil





When the currents are flowing in opposite directions the z-component of the magnetic field on the symmetry axis at a distance *z* > 0 from the midpoint between the centers of the two coils is

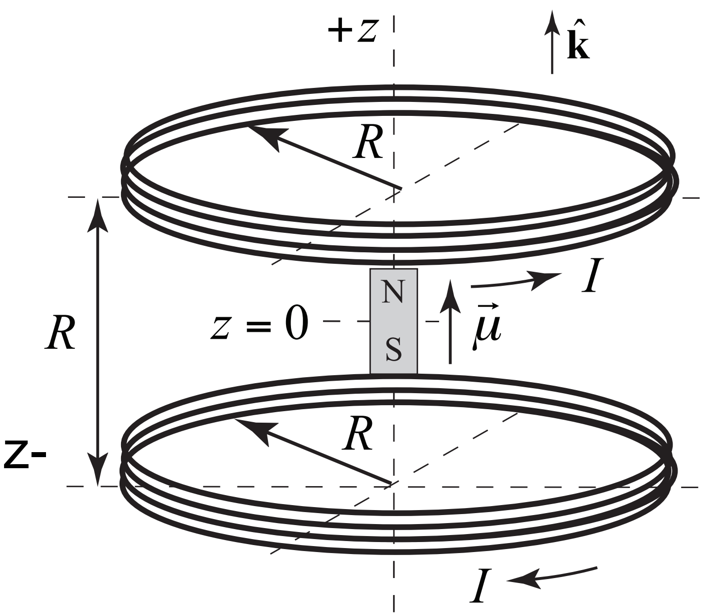
$$\vec{\mathbf{B}}(z) = \vec{\mathbf{B}}_{\text{upper}}(z) + \vec{\mathbf{B}}_{\text{lower}}(z)$$

$$= \frac{\mu_0 NIR^2}{2}\left[\frac{1}{[(z - R/2)^2 + R^2]^{3/2}} - \frac{1}{[(z + R/2)^2 + R^2]^{3/2}}\right]\hat{\mathbf{k}}$$

CQ: Dipole in Anti-Helmholtz Coil

A dipole is initially pointing along the positive *z*-direction and located at the center of an anti-Helmholtz coil (*z* = 0). Which of the following statements is true? Recall the force on a dipole is

$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$



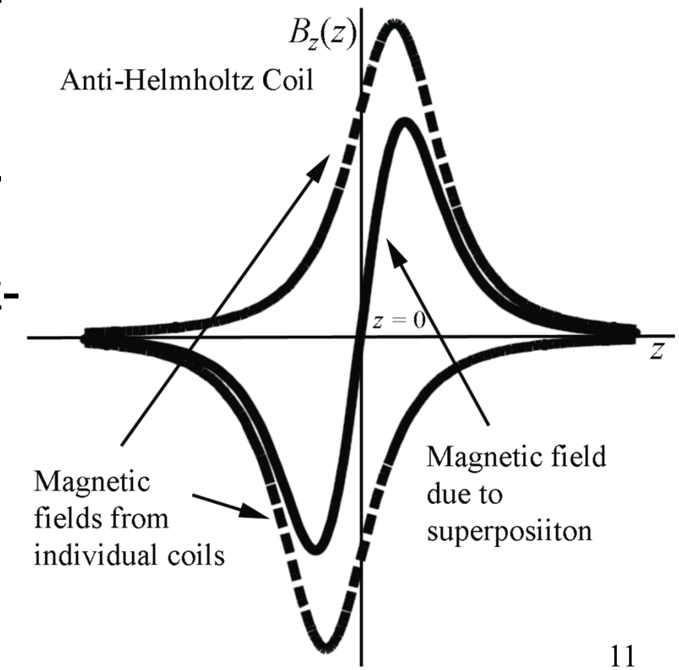
1. The dipole experiences a force in the positive z-direction but zero torque.

2. The dipole experiences a force in the negative z-direction but zero torque.

3. The dipole experiences neither force nor torque.

4. The dipole experiences a force in the negative z-direction and a non-zero torque.

5. The dipole experiences a force in the positive z-direction and a non-zero torque.



Exploration 2 Objectives

- In this exploration we would like you to investigate:

- the magnetic field associated with a variety of coils connections;

- the torque on the magnetic dipole and direction of the force on the dipole when the dipole is placed along different points along the central axis for a two coil connections, the “Helmholtz” and “Anti-Helmholtz” configurations.

Exploration 2 Start-up Instructions

Open Week 7 chapter and select Exploration 2 Helmholtz Coil.

Move to Objective 1 and start the exploration.