Experiment 6: Wireless Energy Transfer; Displacement Current

W13D2
Announcements

PS 11 due Week 14 Friday at 9 pm in boxes outside 26-152

Week 13 Prepset due online Friday 8:30 am

Sunday Tutoring 1-5 pm in 26-152
Outline

Transformers

Experiment 6: Wireless Energy Transfer

Displacement Current
Transformer

Equal flux through each turn:

\[ \mathcal{E}_p = N_p \frac{d\Phi}{dt}; \quad \mathcal{E}_s = N_s \frac{d\Phi}{dt} \]

\[ \frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p} \]

\( N_s > N_p \): step-up transformer

\( N_s < N_p \): step-down transformer
Transformer: Current

Ideal transformer has no power loss

\[ P_p = \mathcal{E}_p I_p; \quad P_s = \mathcal{E}_s I_s \]

\[ P_p = P_s \Rightarrow \mathcal{E}_p I_p = \mathcal{E}_s I_s \]

\[ \frac{I_s}{I_p} = \frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s} \]

\[ N_s > N_p: \text{ step-down current} \]
\[ N_s < N_p: \text{ step-up current} \]
Demonstrations:

26-152 One Turn Secondary: Nail H10

26-152 Many Turn Secondary: Jacob’s Ladder H11

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H 10&show=0

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H 11&show=0
CQ: Residential Transformer

1. House=Left, Line=Right
2. Line=Left, House=Right
3. I don’t know

If the transformer in the can looks like the picture, how is it connected?
Marconi Coil H12

http://tsgphysics.mit.edu/front/?page=demo.php&letnum=H_12&show=0
Transmission of Electric Power

Power loss can be greatly reduced if transmitted at high voltage.
Example: Transmission lines

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V.

(a) \[ I = \frac{P}{V} = \frac{1.2 \times 10^5}{2.4 \times 10^2} = 500\text{A} \]

\[ P_L = I^2R = (500\text{A})^2(0.40\Omega) = 100\text{kW} \]

83% loss!!

(b) \[ I = \frac{P}{V} = \frac{1.2 \times 10^5}{2.4 \times 10^4} = 5.0\text{A} \]

\[ P_L = I^2R = (5.0\text{A})^2(0.40\Omega) = 10\text{W} \]

0.0083% loss
Voltage Drop Across Transmission Line

The voltage drop across the transmission line is

\[ V = IR = (5 \text{ A})(0.4 \Omega) = 2 \text{ V} \]

Therefore the power loss across the resistor is

\[ P = I^2R = \frac{V^2}{R} = \frac{(2.0 \text{ V})^2}{0.4 \Omega} = 10 \text{ W} \]

The voltage drop across the transmission line is not 24,000 V.
Experiment 6: Wireless Energy Transfer
James Clerk Maxwell
Maxwell’s Equations: Is There Something Missing?

\[ \oint_S \mathbf{E} \cdot \mathbf{n} \, dA = \frac{1}{\varepsilon_0} \iiint_V \rho \, dV \]  
(Gauss's Law)

\[ \oint_S \mathbf{B} \cdot \mathbf{n} \, dA = 0 \]  
(Magnetic Gauss's Law)

\[ \oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \oint_S \mathbf{B} \cdot \mathbf{n} \, dA \]  
(Faraday's Law)

\[ \oint_C \mathbf{B} \cdot d\mathbf{s} = \mu_0 \oint_S \mathbf{J} \cdot \mathbf{n} \, dA \]  
(Ampere's Law quasi-static)
Maxwell’s Equations
One Last Modification: Displacement Current
1. Find an expression for the electric field between the plates in terms of the charge Q. Neglect edge effects.

2. Calculate the flux of the electric field between the plates:

\[ \int \int \vec{E} \cdot \hat{n} \, dA \]

3. Calculate the product of the \( \varepsilon_0 \) with the time derivative of the flux of the electric field between the plates

\[ \varepsilon_0 \frac{d\Phi_E}{dt} \]
Parallel Plate Capacitor

Gauss’s Law: \[ E = \frac{Q}{A \varepsilon_0} \]

Flux of electric field: \[ \Phi_E = EA = \frac{Q}{\varepsilon_0} \]

Time changing electric field: \[ \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ}{dt} = I \]
Ampere’s Law: Capacitor

Consider a charging capacitor: Use Ampere’s Law to calculate the magnetic field just above the top plate.

\[ \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} \]

\[ I_{enc} = \iint \vec{J} \cdot \hat{n} \, da \]

1) Surface \( S_1 \): \( I_{enc} = I \)
2) Surface \( S_2 \): \( I_{enc} = 0 \)

What’s Going On?
Displacement Current

We don’t have current between the capacitor plates but we do have a changing $E$ field. Can we “make” a current out of that?

$$E = \frac{Q}{\varepsilon_0 A} \implies Q = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

This is called the displacement current.

It is not a flow of charge but proportional to changing electric flux
Displacement Current:

\[ I_{\text{dis}} = \varepsilon_0 \frac{d}{dt} \int \int_S \vec{E} \cdot \hat{n} \, da = \varepsilon_0 \frac{d\Phi_E}{dt} \]

If surface \( S_2 \) encloses all of the electric flux due to the charged plate then \( I_{\text{dis}} = I \).
Maxwell-Ampere’s Law

\[ \oint_{C} \mathbf{B} \cdot d\mathbf{s} = \mu_0 \iiint_{S} \mathbf{J} \cdot \mathbf{n} \, da + \mu_0 \varepsilon_0 \frac{d}{dt} \iiint_{S} \mathbf{E} \cdot \mathbf{n} \, da \]

\[ = \mu_0 (I_{enc} + I_{dis}) \]

“flow of electric charge”

\[ I_{enc} = \iiint_{S} \mathbf{J} \cdot \mathbf{n} \, da \]

“changing electric flux”

\[ I_{dis} = \varepsilon_0 \frac{d}{dt} \iiint_{S} \mathbf{E} \cdot \mathbf{n} \, da \]
The figures above shows a side and top view of a capacitor with charge $Q$ and electric and magnetic fields $E$ and $B$ at time $t$. At this time the charge on the upper plate $Q$ is:

1. Increasing in time
2. Constant in time.
3. Decreasing in time.
A circular capacitor of spacing $d$ and radius $R$ is in a circuit carrying a steady current $I$ shown at time $t$. At time $t = 0$, the plates are uncharged.

a) Using Gauss’s Law, find a vector expression for the electric field $\mathbf{E}(t)$ at $P$ vs. time $t$ (mag. & dir.)

b) Find a vector expression for the magnetic field $\mathbf{B}(t)$ at $P$
Maxwell’s Equations

\[ \oint_{\partial V} \mathbf{E} \cdot \mathbf{n} \, dA = \frac{1}{\varepsilon_0} \iiint_{V} \rho \, dV \]  
(Gauss's Law)

\[ \oint_{\partial S} \mathbf{B} \cdot \mathbf{n} \, dA = 0 \]  
(Magnetic Gauss's Law)

\[ \oint_{C} \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \oint_{S} \mathbf{B} \cdot \mathbf{n} \, dA \]  
(Faraday's Law)

\[ \oint_{C} \mathbf{B} \cdot d\mathbf{s} = \mu_0 \oint_{S} \mathbf{J} \cdot \mathbf{n} \, dA + \mu_0 \varepsilon_0 \frac{d}{dt} \oint_{S} \mathbf{E} \cdot \mathbf{n} \, dA \]  
(Maxwell-Ampere's Law)