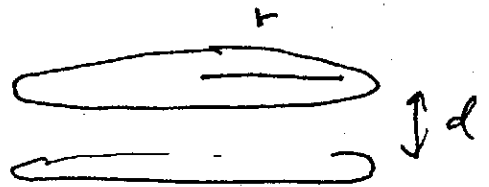


Experiment MF Analysis:

I ran 3 trials with current running in opposite directions producing a repulsive force between the magnets.

From the lab, we approximate this force (see problem 6) by

$$F_{10 \text{ coil}} = \frac{\mu_0 n_1 n_2 i^2 r}{d}$$



where $n_1 = 10$, $n_2 = 38$

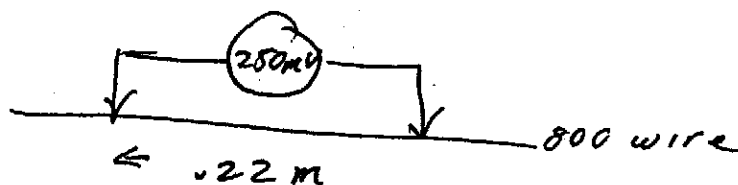
$$r = 3.36 \text{ cm} \pm 0.15 \text{ cm}$$

$$d = 7 \text{ mm} \pm 1.5 \text{ mm}$$

When I placed 2 cm x 2 cm pieces of Al foil directly above the center of the coil, the

$F_{\text{grav}} = n \rho g A t$, $n = \#$ of foils

of the foil acted to counteract the repulsive force of the coils. I turned up the voltage and measured the current across the 800 wire



$$\bar{i} = \frac{V_{\text{measured}}}{R_{\text{wire}}}$$

where $R_{\text{wire}} = (1.02 \frac{\Omega}{\text{m}})(0.22 \text{ m}) = 0.2244 \Omega$

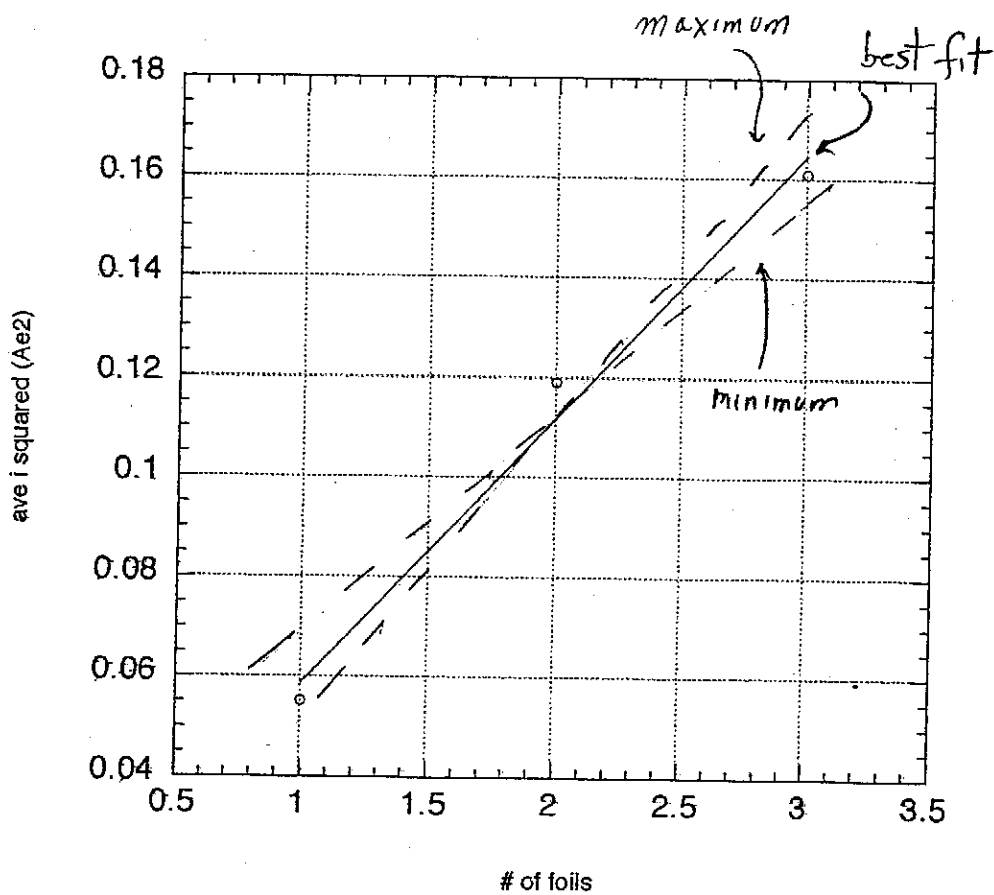
My data and plot of i^2 vs n is shown.

Expt MF data and graph:
4/7/00

	# of foils	ave voltage (mV)	ave current (A)	ave i squared (Ae2)
0	1	52.700	0.235	0.0551
1	2	77.300	0.344	0.119
2	3	90.000	0.401	0.161

	# of foils	trial 1 (mV)	trial 2 (mV)	trial 3 (mV)	ave (mV)
0	1	52.0	51.0	55.0	52.7
1	2	78.0	80.0	74.0	77.3
2	3	90.0	88.0	92.0	90.0

$$\chi^2 = 0.0058 + 0.0529R = 0.993$$



When the forces balanced,

$$\frac{\mu_0 n_1 n_2 \bar{i}^2 r}{d} = \pi \rho g A t$$

$$\bar{i}^2 = \frac{\rho g A t d}{\mu_0 n_1 n_2 r}$$

$$\alpha \equiv \text{slope} = \frac{\rho g A t d}{\mu_0 n_1 n_2 r}$$

From by graph, the $\alpha \equiv \text{slope} = .0529 \frac{A^2}{\#}$

I drew maximum and minimum lines which fit the data and found

$$(\alpha_{\max}) = .060 \frac{A^2}{m^2} \quad (\alpha_{\min}) = .043 \frac{A^2}{m^2}$$

So I estimate the error of my slope

to be

$$\frac{\Delta \alpha}{\alpha} \approx .16$$

where I averaged $\frac{(\alpha_{\max} - \alpha) + (\alpha - \alpha_{\min})}{2} = \Delta \alpha = .0087 \frac{A^2}{\#^2}$

Since $\mu_0 = \frac{1}{(\text{slope})} \frac{\rho g A t d}{n_1 n_2 r}$ where

$$\rho = 2.7 \times 10^3 \text{ kg/m}^3, \quad A = 4 \times 10^{-4} \text{ m}^2, \quad t = 1.8 \times 10^{-5} \text{ m}$$

$$d = 7 \times 10^{-3} \text{ m}, \quad n_1 = 10, \quad n_2 = 38, \quad r = 3.36 \times 10^{-2} \text{ m}$$

$$\mu_0 = \frac{1}{(.0529 \frac{A^2}{\#})} \frac{(2.7 \times 10^3 \frac{\text{kg}}{\text{m}^3})(9.8 \frac{\text{m}}{\text{s}^2})(4 \times 10^{-4} \text{ m}^2)(1.8 \times 10^{-5} \text{ m})(7 \times 10^{-3} \text{ m})}{(10)(38)(3.36 \times 10^{-2} \text{ m})}$$

$$= 1.97 \times 10^{-6} \frac{\text{tesla} \cdot \text{m}}{\text{A}}$$

The accepted value for μ_0 is

$$(\mu_0)_{th} = \frac{4\pi \times 10^{-7} \text{ T}\cdot\text{m}}{\text{A}} = 1.26 \times 10^{-6}$$

Now the biggest error is in the distance between the coils

$$\frac{\Delta d}{d} = \frac{1.5 \text{ mm}}{7 \text{ m}} = .21$$

The approximation to the B field

$$\frac{\Delta B}{B} \approx .05 \quad (\text{see A.P. French: note on MF})$$

So

$$\begin{aligned} \frac{\Delta \mu_0}{\mu_0} &= \left(\left(\frac{\Delta \alpha}{\alpha} \right)^2 + \left(\frac{\Delta d}{d} \right)^2 + \left(\frac{\Delta B}{B} \right)^2 \right)^{1/2} \\ &= \left((0.16)^2 + (0.21)^2 + (0.05)^2 \right)^{1/2} = .26 \end{aligned}$$

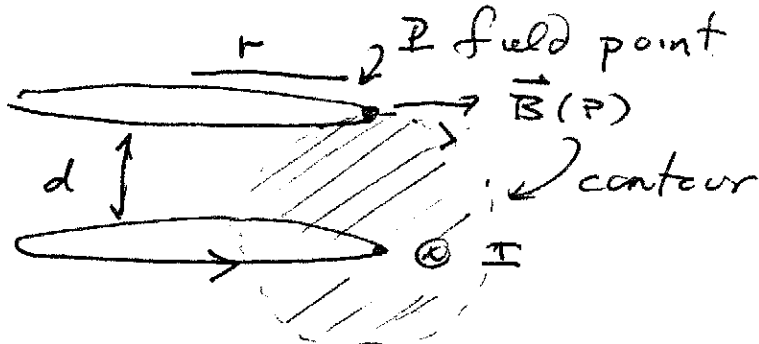
$$\begin{aligned} \frac{(\mu_0)_{\text{expt}} - (\mu_0)_{th}}{(\mu_0)_{th}} &= \frac{1.97 \times 10^{-6} - 1.26 \times 10^{-6}}{1.26 \times 10^{-6}} \\ &= .56 \end{aligned}$$

The discrepancy between the experimental result and the theoretical results suggest that there is a systematic error. The most likely place is the thickness of the foil $t = 1.8 \times 10^{-5} \text{ m}$.

Problem 2

a)

$N=10$



$N=38$

From Ampere's Law

$$\frac{\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}}{B 2\pi d = \mu_0 (38) (I)}$$

$$B = \frac{38 \mu_0 I}{2\pi d} = \frac{(38)(2 \times 10^{-7} \frac{T \cdot m}{A})(0.5 A)}{(5 \times 10^{-3} m)}$$

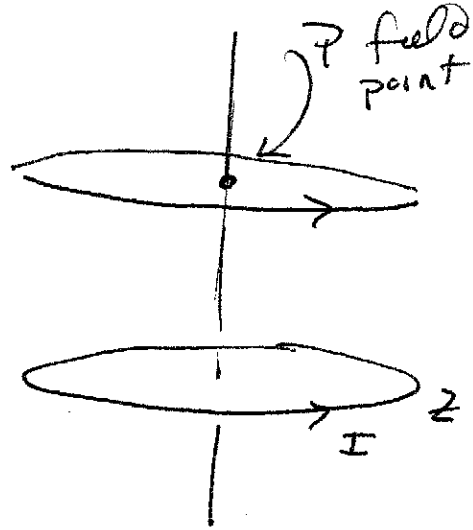
where $\frac{\mu_0}{2\pi} = 2 \times 10^{-7} \frac{T \cdot m}{A}$

$$B = 7.6 \times 10^{-4} T = 7.6 \text{ gauss}$$

In this calculation, we are approximating the magnetic field of the $N=38$ turn coil as the field of a long straight wire for field points with $d \ll r$

b)

$N=10$



$N=38$

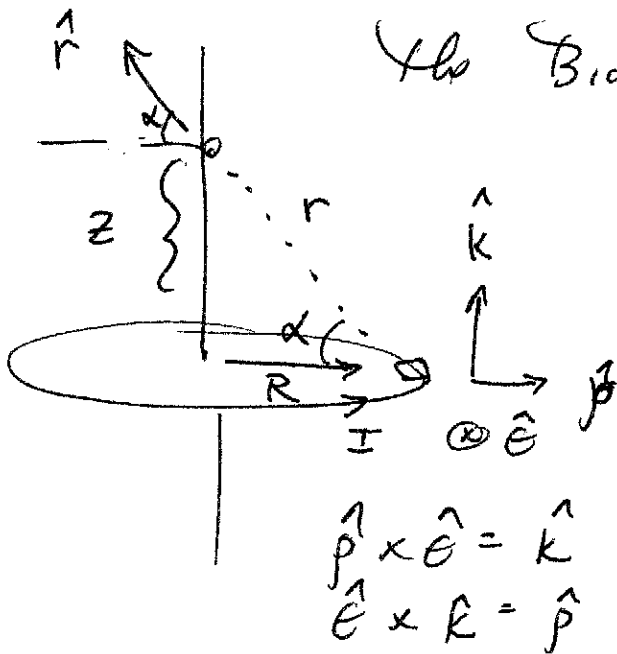
$z=0$

When the coils are in series the current flows in the same direction

$$\vec{B}(P) = \vec{B}_{38}(z) + \vec{B}_{10}(\text{center}) \text{ superposition}$$

So let's calculate the $\vec{B}_{38}(z)$ using

the Biot-Savart Law



$$\vec{B} = \int_{\text{coil}} \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{r}}{r^2}$$

$$I d\vec{l} = N_{38} I R d\theta \hat{e}$$

$$\vec{r} = \cos\alpha (-\hat{p}) + \sin\alpha \hat{k}$$

$$r = (R^2 + z^2)^{1/2}$$

$$\begin{aligned} \hat{p} \times \hat{e} &= \hat{k} \\ \hat{e} \times \hat{k} &= \hat{p} \end{aligned}$$

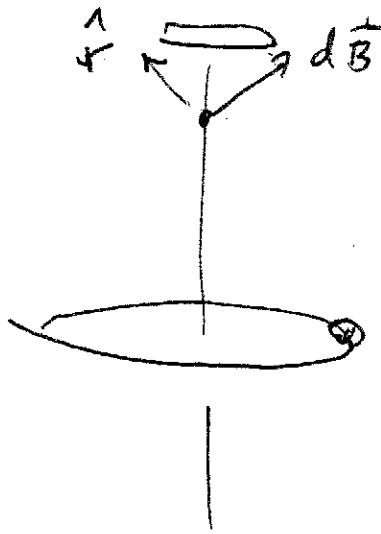
$$\cos\alpha = \frac{R}{r}, \quad \sin\alpha = \frac{z}{r}$$

$\theta = 0$ to 2π

$$\vec{B} = \int_0^{2\pi} \frac{\mu_0}{4\pi} \frac{N_{38} I R d\theta \hat{e} \times (-\cos\alpha \hat{p} + \sin\alpha \hat{k})}{r^2}$$

$\theta = 0$

$$= \frac{\mu_0}{4\pi} \frac{N_{38} I R}{r^2} \left(\int_0^{2\pi} d\theta \cos\alpha \hat{k} + \int_0^{2\pi} d\theta \sin\alpha \hat{p} \right)$$



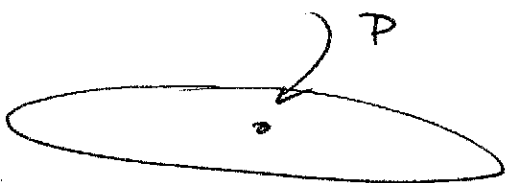
As we go around the circle, $d\vec{B}$ sweeps out a cone. The tangential component sums to zero, leaving only a vertical non-zero component.

$$\vec{B}(z) = \frac{\mu_0}{4\pi} N_{38} I R \int_0^{2\pi} \frac{d\theta R}{r^3} \hat{k}$$

$$= \frac{\mu_0}{4\pi} N_{38} \frac{I R^2}{r^3} 2\pi \hat{k}$$

$$\vec{B}_{38}(z) = \frac{\mu_0}{4\pi} N_{38} \frac{I R^2}{(R^2 + z^2)^{3/2}} 2\pi \hat{k}$$

For the 10-turn coil



$$\vec{B}_{10}(\text{center}) = \frac{\mu_0}{2} N_{10} \frac{I}{R} \hat{k}$$

$$\vec{B}^{\text{total}} = \vec{B}_{38}(z) + \vec{B}_{10}(\text{center})$$

$$= \left(\frac{\mu_0 N_{38} I R^2}{2 (R^2 + z^2)^{3/2}} + \frac{\mu_0 N_{10} I}{2 R} \right) \hat{k}$$

$$= \left(\frac{\mu_0 I}{2} \right) \left(\frac{N_{38} R^2}{(R^2 + z^2)^{3/2}} + \frac{N_{10}}{R} \right) \hat{k} \quad , \quad \frac{\mu_0}{2\pi} = 2 \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}$$

$$= (2 \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}) (\pi) (0.5 \text{ A}) \left(\frac{(38)(6 \times 10^{-2} \text{ m})^2}{(6 \times 10^{-2} \text{ m})^2 + (0.5 \times 10^{-2} \text{ m})^2} + \frac{10}{6 \times 10^{-2} \text{ m}} \right) \hat{k}$$

where $z = 0.5 \times 10^{-2} \text{ m}$

$$\vec{B} = 2.5 \times 10^{-4} \text{ T} \hat{k}$$

$$= (2.5 \text{ gauss}) \hat{k}$$