**News**

- Quiz #2: Monday, 3/14, 10AM
- Same procedure as for quiz 1
  - Review in class Fri, 3/11
  - Evening review, Fri, 3/11, 6-8PM, 54-100
  - 2 practice quizzes (+ practice problems)
  - Formula sheet

**Charge and Potential**

- For given geometry, Potential and Charge are proportional
- Define \( Q = C V \) \( \Rightarrow C \) is Capacitance
- measured in \([F] = [C/V] : \text{Farad}\)
- \( C \) tells us, how easy it is to store charge on it \( (V = Q/C)\)
Capacitance

\[ Q = \frac{C}{V(a) - V(b)} = \frac{Q}{V_0} \]

C bigger -> Can store more Charge!

Capacitor

- Def: Two conductors separated by insulator
- Charging capacitor:
  - take charge from one of the conductors and put on the other
  - separate + and - charges

Parallel Plate Capacitor

\[ C = \frac{Q}{A \epsilon_0} = \frac{Q}{A \epsilon_0 d} \]

To store lots of charge
- make A big
- make d small

Energy stored in Capacitor

\[ W_{tot} = \int_{Q_{initial}}^{Q_{final}} V \ dq = \int_0^Q V \ dq \]

\[ = \int_0^Q \frac{q}{C} \ dq = \frac{1}{C} \int_0^Q q \ dq \]

\[ = \frac{1}{2} \frac{Q^2}{C} \]

- Work \( W = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \ C V^2 \) needed to charge capacitor
- Energy conserved
- But power can be amplified
  - Charge slowly
  - Discharge very quickly
where is the energy stored?

- Energy is stored in Electric Field
  \[
  U_{\text{stored}} = \frac{1}{2} CV^2 = \frac{1}{2} (\varepsilon_0 \frac{A}{d})(E^2 d)^2 = \frac{1}{2} \varepsilon_0 E^2 \text{Volume}
  \]

- \( E^2 \) gives Energy Density:
- \( U/\text{Volume} = \frac{1}{2} \varepsilon_0 E^2 \)

Dielectrics

- Parallel Plate Capacitor:
  - \( C = \varepsilon_0 A/d \)
  - Ex. \( A = 1\, \text{m}^2, \, d=0.1\, \text{mm} \)
  - -> \( C \approx 0.1\, \mu\text{F} \)
- How can one get small capacitors with big capacity?

In your toolbox:

<table>
<thead>
<tr>
<th>Side</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cm</td>
<td>C = 1000 ( \mu \text{F} )</td>
</tr>
</tbody>
</table>

Dielectric Demo

- Start w/ charged capacitor
  - \( d \) big -> \( C \) small -> \( V \) large
- Insert Glass plate
- Now \( V \) much smaller
- \( C \) bigger
  - But \( A \) and \( d \) unchanged!
- Glass is a **Dielectric**
Microscopic view

Polarization $\vec{P} = \text{const.}$. $\vec{E} = \varepsilon_0 \times \vec{E}$

Dielectric Constant

- Dielectric reduces field $E_0$ ($K > 1$)
  - $E = \frac{1}{K} E_0$
- Dielectric increases Capacitance
  - $C = \frac{Q}{V} = \frac{Q}{(E d)} = K \frac{Q}{(E_0 d)}$
- This is how to make small capacitors with large $C$!

Electric Current

- We left Electrostatics
  - Now: Charges can move in steady state
- Electric Current $I$:
  - $I = \frac{dQ}{dt}$
  - Net amount of charge moving through conductor per unit time
- Units:
  - $[I] = \frac{C}{s} = A$ (Ampere)
**Demo I**

Ions discharge
Electroscope

Charged Ions

**Demo II**

Glass

Light Bulb

Voltage source

Molten glass: Charge carriers become mobile -> Current flows -> Bulb lights up!

**Demo III**

Distilled Water

Light Bulb

Add NaCl: Dissociates into Na\(^+\) and Cl\(^-\)
Charge carriers are available -> Current flows -> Bulb lights up

**Demo IV**

Liquid Nitrogen
(T ~ -200°C)

Light bulb (bright)

Voltage source

Wire cold -> less resistance -> more current -> bulb burns brighter
**Resistivity**

- Interplay of scattering and acceleration gives an average velocity $v_D$
- $v_D$ is called ‘Drift velocity’
- How fast do the electrons move?
  - Thermal speed is big: $v_{th} \sim 10^6 \text{ m/s}$
  - Drift velocity is small: $v_D \sim 10^{-3} \text{ m/s}$
- All electrons in conductor start to move, as soon as $E > 0$

**Resistance**

- Define $R = V/I$ : Resistance
- $R = \rho L/A$ for constant cross section $A$
- $R$ is measured in Ohm [$\Omega$] = [V/A]
- Resistivity $\rho$ is property of material (e.g. glass)
- Resistance $R$ is property of specific conductor, depending on material ($\rho$) and geometry

**Ohm’s law**

$$V = R I$$

- Conductor is ‘Ohmic’, if $R$ does not depend on $V,I$
- For real conductors, that is only approximately true (e.g. $R = R(T)$ and $T = T(I)$)
- Approximation
- valid for resistors in circuits
- not valid for e.g. light bulbs

**Electric Power**

- Use moving charges to deliver power
  - $\text{Power} = \text{Energy/time} = dW/dt$
  - $dW/dt = (dqV)/dt = dq/dt V = I V = I^2 R = V^2/R$
Electric circuits

Resistor

Capacitor

Source of EMF

Electromotive Force EMF

- Def: \( \xi = \text{Work/unit charge} \)
- \( \xi \) is ‘Electromotive Force’ (EMF)
- Units are \([V]\)

Internal resistance

- Sources of EMF have internal resistance \( r \)
- Can’t supply infinite power

Battery

\[ V_{ab} = \xi - I r \]
= IR

-> \[ I = \frac{\xi}{(r+R)} \]

Electric Circuits

Resistors in series

\[ V_{ac} = V_{ab} + V_{ac} = I R_1 + I R_2 = I (R_1 + R_2) \]
= \( I \) \( R_{eq} \) for \( R_{eq} = (R_1 + R_2) \)
Electric Circuits

Resistors in parallel

\[ V_a - R_1 - I - V_b \]

\[ R_2 \]

\[ I = I_1 + I_2 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} = \frac{V_{ab}}{R_{eq}} \]

\[ \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

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### Demo

- Two capacitors in parallel
- \( V_{56} = V_{23} = V_{14} \) (after capacitor is charged)
- \( \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = V_{14} \)
- \( Q_{tot} = Q_1 + Q_2 \)
- \( C_{tot} = (Q_1 + Q_2) / V_{14} = C_1 + C_2 \)

Capacitors in parallel -> Capacitances add!

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\[ EMF = V_{12} + V_{34} + V_{56} + V_{78} + V_{910} \]
In general, $\sum V_j = 0$

Kirchoff’s rules

- Kirchoff’s rules allow us to calculate currents for complicated DC circuits
- Main difficulty: Signs!
- Rule for resistors:

$$\Delta V = V_b - V_a = -IR, \text{ if we go in the direction of } I \text{ (voltage drop!)}$$
Example

- Pick signs for $I_1$, $\xi$
- Junction rule
  $I_1 = 1A + 2A = 3A$
- Loop rule (1)
  $12V - 6V - 3A \cdot r = 0$
  $r = 6/3 \Omega = 2 \Omega$
- Loop rule (2)
  $12V - 6V - 1V - \xi = 0$
  $\Rightarrow \xi = 5V$

$\xi$, $I_1$? 3 unknowns

Experiment EF

\[ F_{on,foil} = Q_{foil} \tilde{E}_{top} \]

\[ \tilde{E}_{top} = \frac{\sigma}{2\varepsilon_0} = \frac{Q/A_{wash}}{2\varepsilon_0} (-\dot{y}) \]

\[ Q_{foil} = -\frac{A_{foil}}{A_{wash}} \]

\[ Q = CV = \varepsilon_0 A/d V \]

\[ \Rightarrow F_{on,foil} = -Q \frac{A_{foil}}{A_{wash}} \frac{Q}{A_{wash}2\varepsilon_0} (-\dot{y}) \Rightarrow F_{on,foil} = \frac{(\varepsilon_0 V A_{wash}/d)^2 A_{foil}}{2\varepsilon_0} \dot{y} \]

\[ = \frac{Q^2}{A_{wash}^2} \frac{A_{foil}}{2\varepsilon_0} \dot{y} \]

\[ = \frac{\varepsilon_0 V^2}{2d^2} A_{foil} \dot{y} \]