PROBLEM SET 10; 802x; SPRING 2005

1) The differential equation governing an RLC circuit is:

$-Ldi/dt - Ri + q/C = 0$. Using $i = - dq/dt$, we have,

$Ld^2q/dt^2 + Rdq/dt + q/C = 0$.

The differential equation governing a mass on a spring is (with velocity proportional viscous damping):

$md^2x/dt^2 + bdx/dt + Kx = 0$. Here the mass is m, K is the spring constant and b is the coefficient of proportionality between velocity and the viscous retarding force.

Thus: M and L play the same roles; b and R play the same roles; and, K plays the same role as 1/C.

Mv is the momentum that will persist unless changed by a force, and Li is the flux in an inductor that will persist unless changed by an external agent. The kinetic energy stored in motion is $(1/2)\ mv^2$, while energy is stored in the inductor as $(1/2)Li^2$. The resistor is an agent for energy loss at the rate i^2R. Energy is lost to viscocity at the rate bv^2. Energy is stored in a capacitor as $(1/2)q^2/C$ and energy is stored in the spring as $(1/2)Kx^2$.

2) The self-inductance of the circuit causes the current to persist until the voltage developed across the gap acting as a capacitor causes it to stop. Now this gap usually has a very small capacitance and the current, which we have assumed to be large, can charge the gap to a very large voltage. Thus the spark develops when the air brakes down. The energy for the spark comes from the energy stored in the self-inductance of the circuit, $(1/2)Li^2$.

The equilibrium current is $i = V/R = 100/10 = 10$ amps. The energy stored in the inductor is $(1/2Li^2 = (1/2)(1/1000)(100) = 1/20$ joule.

3) a. Compare figure 30.18 and fig 30.6b. Note that points a and b are reversed. Thus, according to equation 30.8, $dI/dt = (Vb – Va)/L = - 1.04V/0.260H = -4$ A/s. Thus, the current is decreasing.
b. From a. we know that $di = (-4A/s)\ dt$. After integrating both sides of the expression with respect to t, we obtain $\Delta I = (-4A/s)\Delta t$ and so $I = (12.0A) – 4A/s * 2s = 4A$.

4) a. $U = P*t = (200W)(24h/day x 3600s/h) = 1.73x10^7J$.
b. $U = ½\ L\ I^2$ and therefore $L = 2U/I^2 = 2\ (1.73x10^7J)/(80A^2) = 5406H$.

5) When switch 1 is closed and switch 2 is open, the loop rule gives $L\ dI/dt + IR = 0$ and therefore $dI/dt = - I\ R/L$. Integrating from I_0 to I on the LHS and 0 to t on the RHS gives $\ln(I/I_0) = - R/L\ t$ and therefore $I(t) = I_0 \exp(-t/(L/R))$